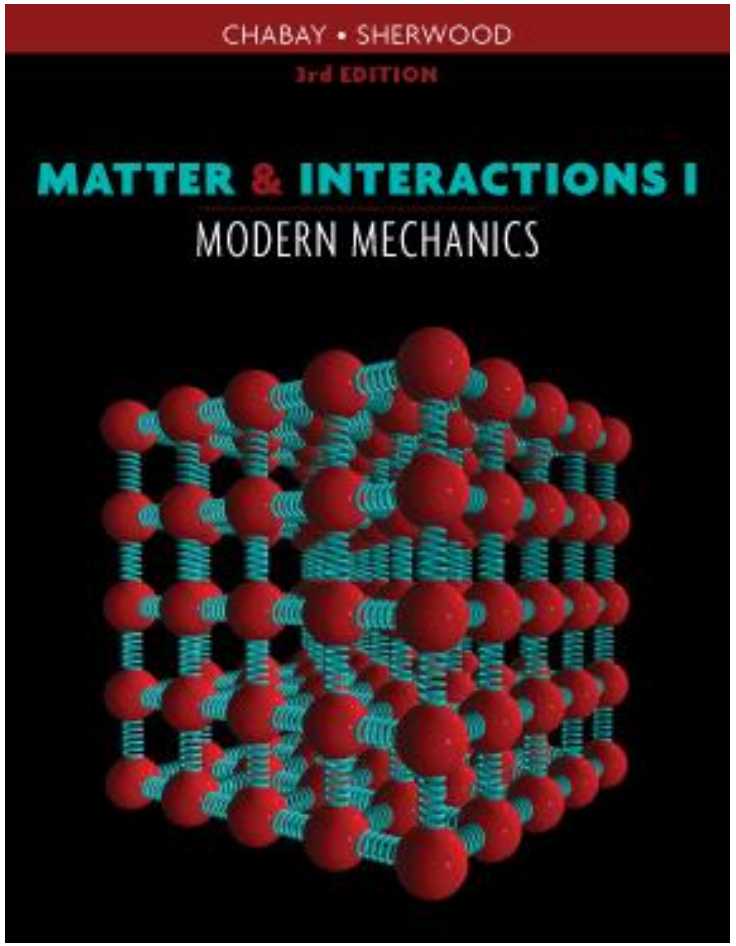


PHY1004W 2013
Modern Mechanics
Part 1

Prof Andy Buffler
Room 503 RW James
andy.buffler@uct.ac.za

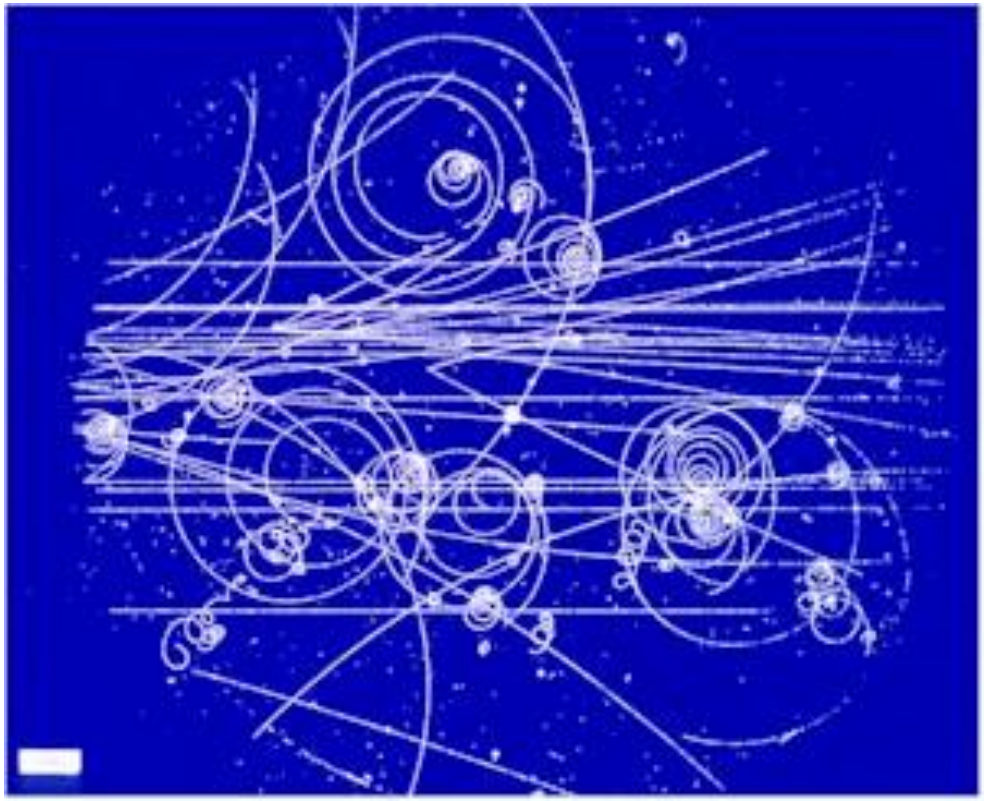
These slides have benefited from significant guidance from the notes of Roger Fearick (UCT Physics) and the resources provided by the textbook authors.

These slides are available on … UCT OpenContent

M&I

Chapter 1

Interactions and Motion

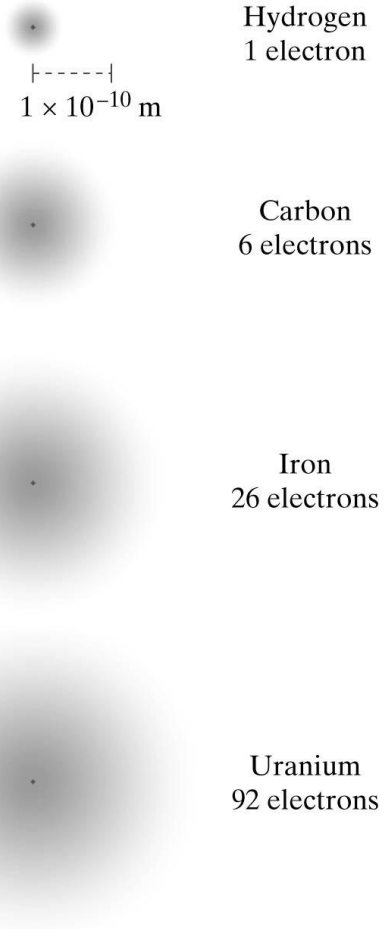


Kinds of matter

The world around us consists of matter.

Atoms & nuclei.
Molecules.
- **Solids**
- **Liquids**
- **Gases**

Rocks, seas, people, planets, stars, galaxies, . . .
Universe



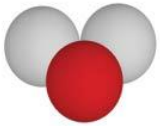


Hydrogen nucleus
1 proton

$\vdash\!\dashv 1 \times 10^{-15}$ m



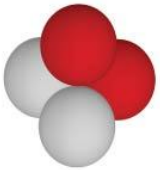
Deuterium nucleus
1 proton + 1 neutron



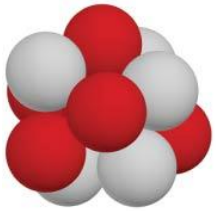
Tritium nucleus
1 proton + 2 neutrons



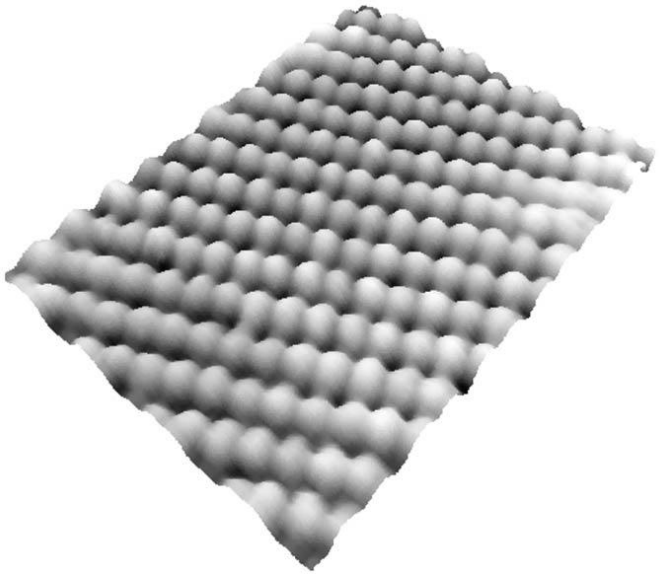
Helium-3 nucleus
2 protons + 1 neutron

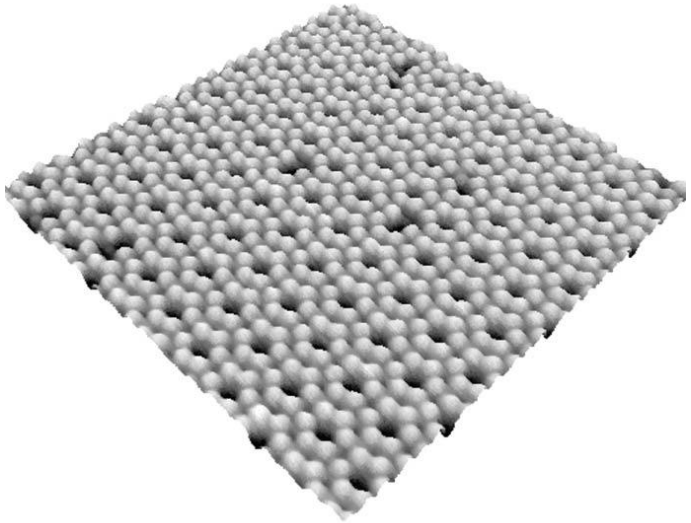


Helium-4 nucleus
2 protons + 2 neutrons



Carbon nucleus
6 protons + 6 neutrons







Cluster
of galaxies



Galaxy

The planet
Jupiter in our
Solar System



See “Powers of Ten”

Very often in physics we need to model the motion of moving bodies. In this course we will in most cases consider the body to be a point particle. We will discuss this later. To start we need to think about how to visually represent the motion of a particle in different ways, before we try to model this motion mathematically. Visual representations can be very useful models since they allow us to make sense of things conceptually.

Consider a box sliding on a frictionless surface. If you were asked to **describe** the motion of the box, there are a number of ways that you could use … these include …
… words … pictures … mathematics …

However, in order to understand a particular situation, this very often requires us to **represent** the situation in a number of different ways … and each of these representations carry very particular types of information about the system in question.

In this course we will build models of systems we want to study by abstracting the key phenomena, and ignoring things that don't make much difference.

"Particle": no extent or internal structure.
"Body": finite extent, internal structure, shape, can be deformed, ...

M&I 3E 1.3

Detecting interactions

Look at the properties of the particle and its motion.

... change in direction.
... change in speed.
... change in velocity.
... change in mass.
... change in shape (of a body), . . .

But what about ... change in position?

Newton's First Law of Motion

Galileo Galilei (1564-1642);
Isaac Newton (1642-1727):

An object moves in a straight line and at constant speed except to the extent that it interacts with other objects.

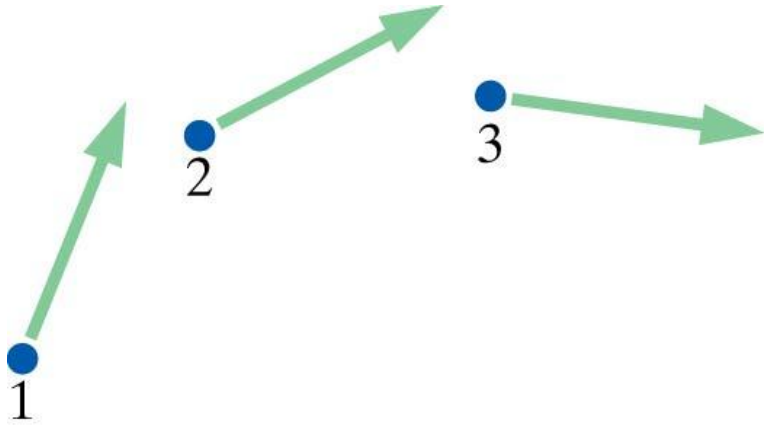
Newton 1 provides the conceptual framework for analysing interactions.

We do not need an interaction to "keep something moving"!

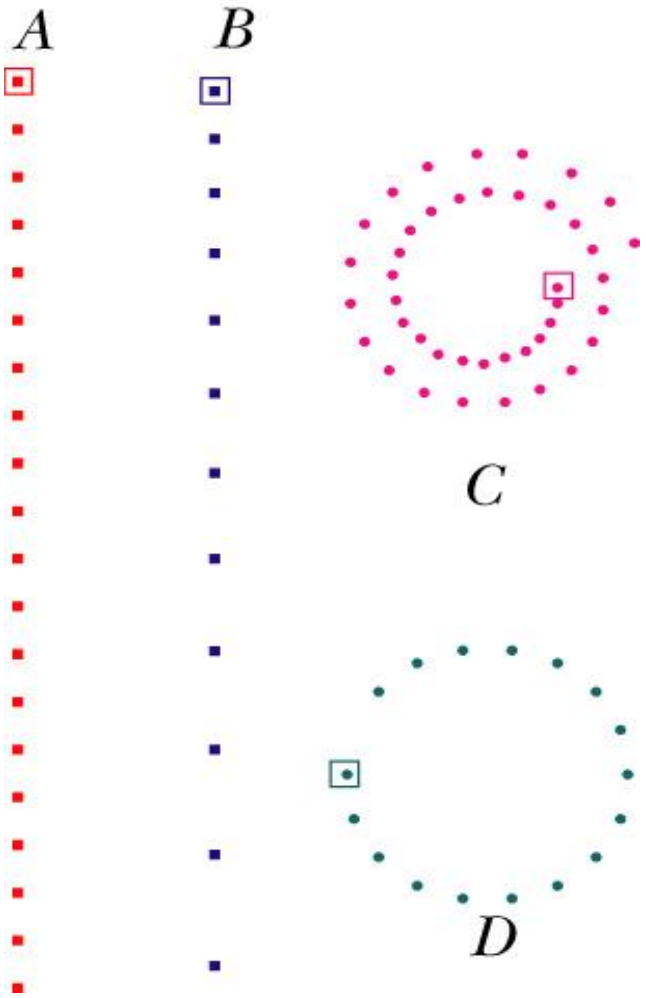
Does Newton I apply in everyday life?

Detecting interactions



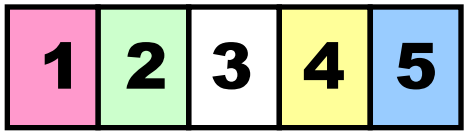






Moving objects left the traces shown at left. The dots were laid down at equal time intervals. Which objects did **not** interact with another object somewhere?

(1) *A*
(2) *B*
(3) *C*
(4) *D*
(5) *A* and *B*



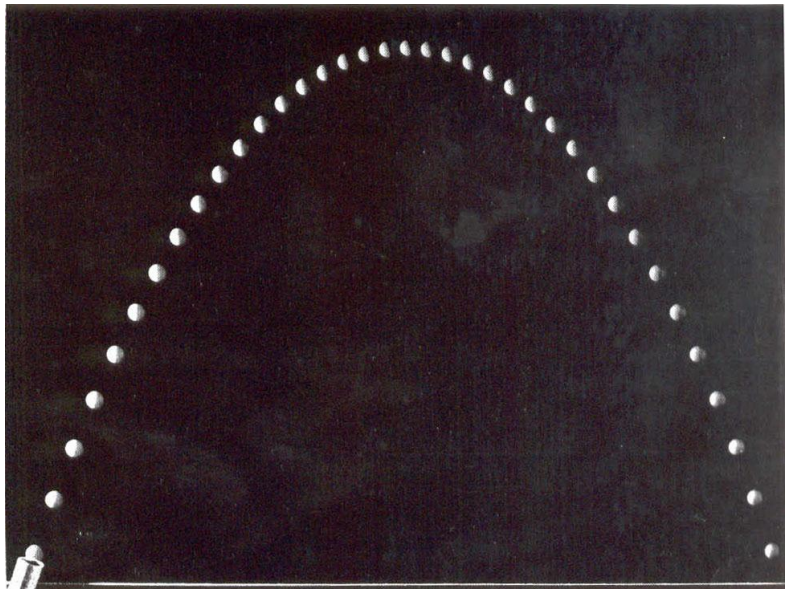
Which of the following can NOT be true for an object moving in a straight line at a constant speed?

(1) Nothing is interacting with the object (it is in interstellar space, far from all other objects).
(2) The object is experiencing a net interaction.
(3) The object is experiencing multiple interactions, and these interactions add up to zero.
(4) The object has no net interaction with the rest of the world.
(5) All are true.

Stroboscopic photographs

An intermediate step between a movie and only drawing the object at its initial and final positions, is to draw in the object at **many** positions along its path.

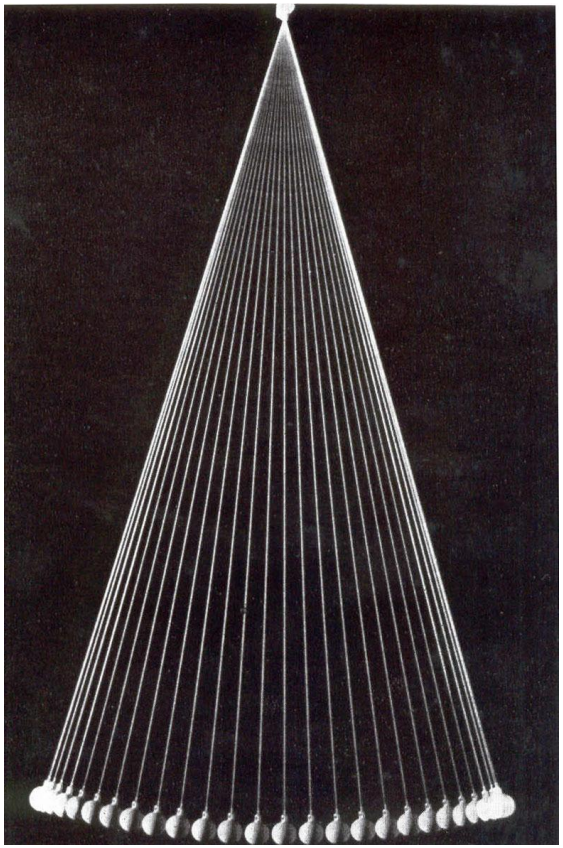
Look at the photograph alongside. It is called a **stroboscopic photograph** since a camera took a photograph ("snapshot") of the ball every Δt apart (synchronized with a strobe light flash) and then all the photographs are displayed on the same frame.



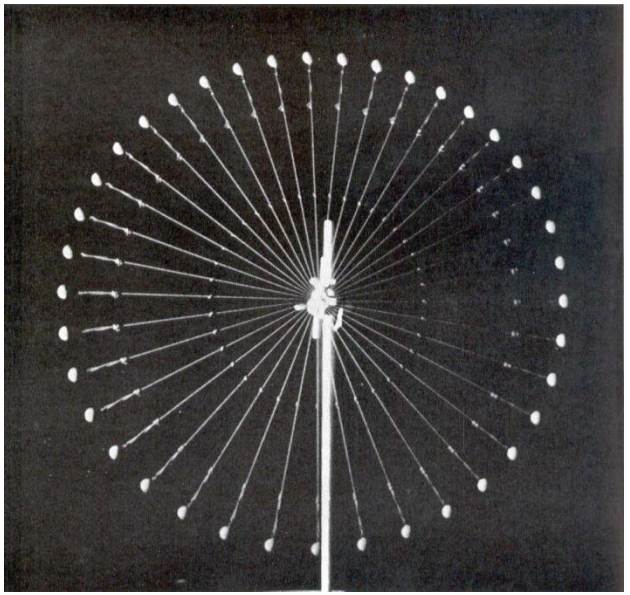
The time interval Δt could be a fraction of a second, of course.

Here are three examples of motion in two dimensions. Explain what is going on in each case.

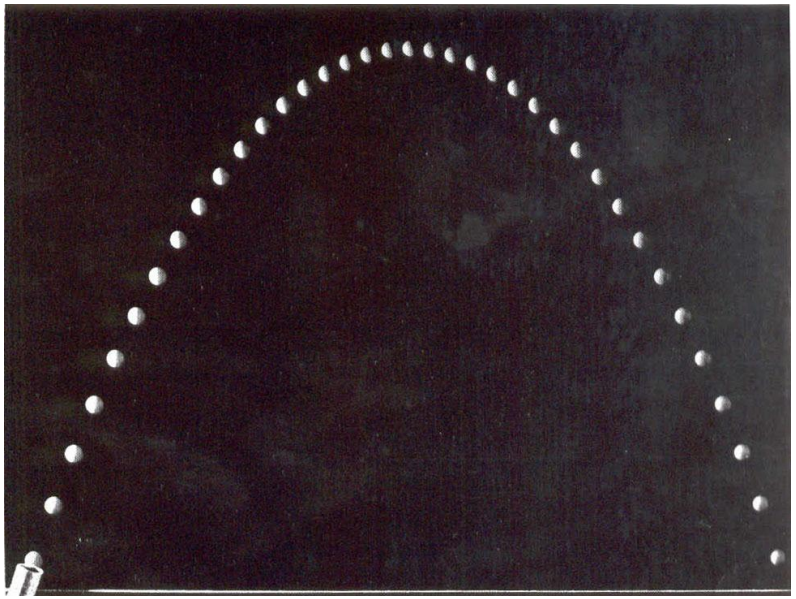
(a)



(b)



(c)



What do we need to describe interactions?

For a quantitative description we need a firm mathematical foundation.
i.e. we need to measure appropriate variables, and relate them mathematically.

- Establish units to express physical measurements.
- Coordinate framework to describe space and time.
- Scalars and vectors to represent physical quantities.
- Express physical description in mathematical terms.

Other indicators of interaction

Change in velocity.
Change in identity.
Change in shape or configuration.
Change in temperature.
Change in ...

Interactions cause change

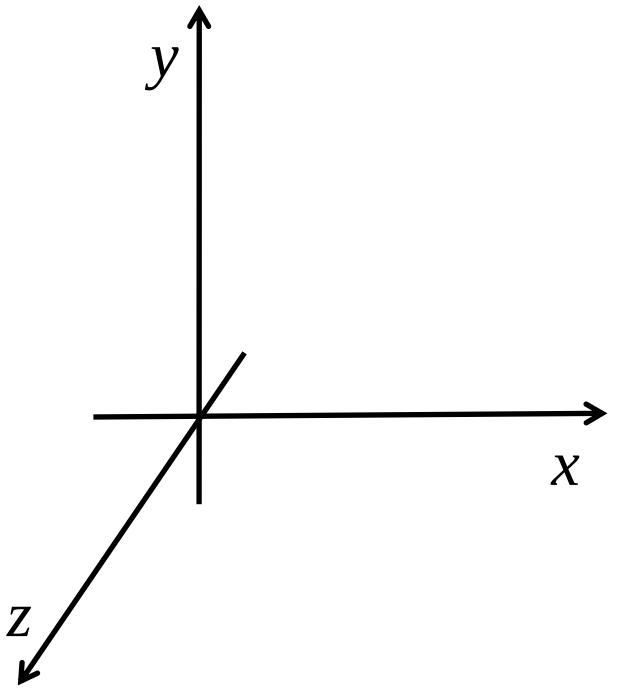
Coordinate systems in 3D

We can represent the position of a point on a line by using a coordinate system.

The coordinates (x, y, z) give the position of a point relative to a set of axes with some origin.

The number of axes required is equal to the dimension of the space.

Axes are most easily handled mathematically when they are at right angles to one another.



Vectors and scalars

It is useful to introduce objects which represent both direction and the magnitude of some quantity.

For example, velocity describes both the speed of something, and the direction it is travelling in.

Such objects are called **vectors**.

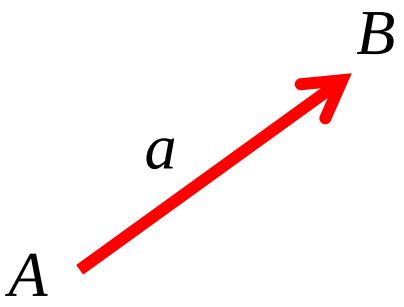
Other vectors: acceleration, momentum, electric field,. . .
A quantity that is represented at a point by one number is known as a **scalar**.

Examples: temperature, energy, mass,. . .

Scalars and vectors may have units.

Vectors: notation

An arrow is used to depict the vector in drawings, with the arrow pointing in the required direction, and the length of the arrow representing the magnitude.



Note that the length is always ≥ 0.

The information is in the direction and length of the vector … we can shift the position of the vector around if we don't change these.

Vectors: notation

Usually, in print, vectors are denoted by **bold Roman type,** e.g. **a** is a vector.

In these notes we will use $\vec{\mathbf{a}}$.

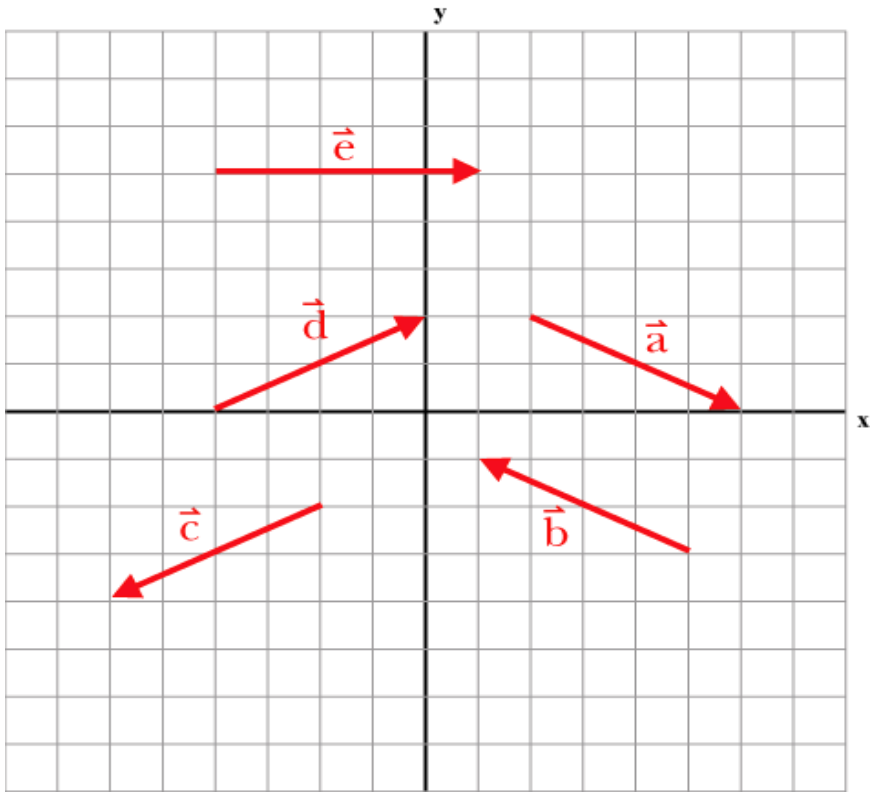
M&I uses an arrow over an italicised (roman) symbol to denote a vector: $\vec{a}$

The magnitude is denoted by the same symbol, in italic, e.g. a is the magnitude of $\vec{a}$.

In script, we may denote a vector as: 

1	2	3	4	5

Which of these arrows represents the vector $< -4, 2, 0 >$?



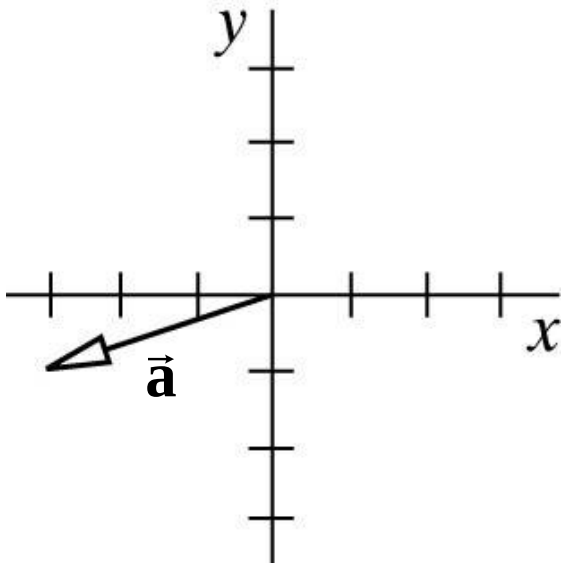
(1) $\vec{\mathbf{a}}$
(2) $\vec{\mathbf{b}}$
(3) $\vec{\mathbf{c}}$
(4) $\vec{\mathbf{d}}$
(5) $\vec{\mathbf{e}}$
(6) none of the above

Position vectors

The coordinates of a point (x, y, z) can be interpreted as a vector $\langle x, y, z \rangle$.

This form of position is known as a **position vector**.

This really represents a displacement from the origin.



$\vec{\mathbf{a}} = \langle -3, -1, 0 \rangle$ metres

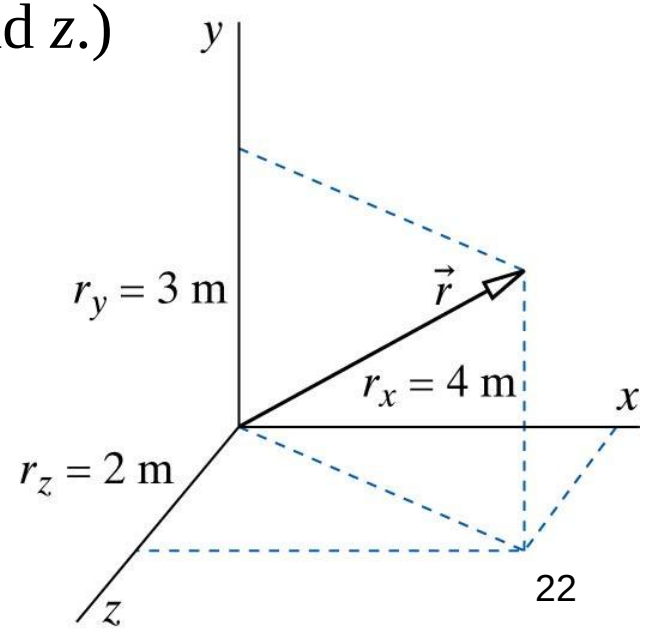
Components of vectors

Each of the numbers in the triplet is known as a **component** of the vector.

A component is the **projection** of a vector on an axis of the coordinate system.

In general, the *x* component represents the difference between the *x* coordinate of the tip of the vector and the *x* coordinate of the tail of the vector. (… similarly for *y* and *z*.)

When using symbols, we label the components with a subscript related to the coordinate axis,
e.g. $\vec{\mathbf{r}} = \langle r_x, r_y, r_z \rangle$



Components of vectors

The components depend on the coordinate system
... the system can be chosen to suit the problem.

The components are signed ("head–tail").

Components have units !

3 components in 3D (e.g. a_x, a_y, a_z).

The components are unchanged by a translation of the axes (but are changed by a rotation).

Vectors: equality

Two vectors are the same if (and only if) all their components are equal.

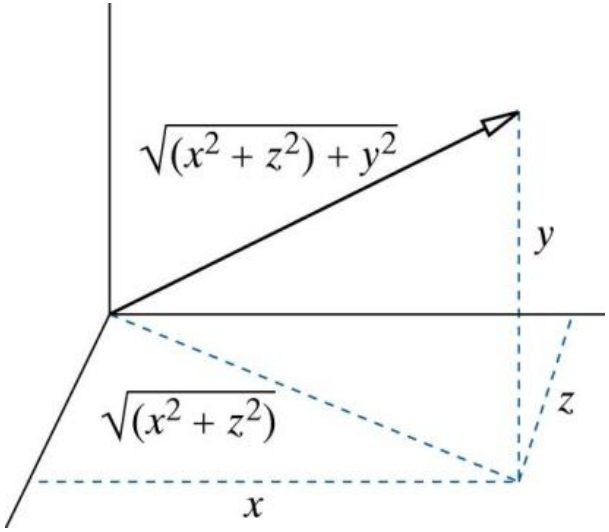
Thus $\vec{\mathbf{a}} = \vec{\mathbf{b}}$ means that $a_x = b_x$, $a_y = b_y$, and $a_z = b_z$

A zero vector has all components zero.

Vectors: the magnitude

The **magnitude** of a vector is the length of the vector.

This can be obtained by Pythagoras' theorem (in 3D if necessary).

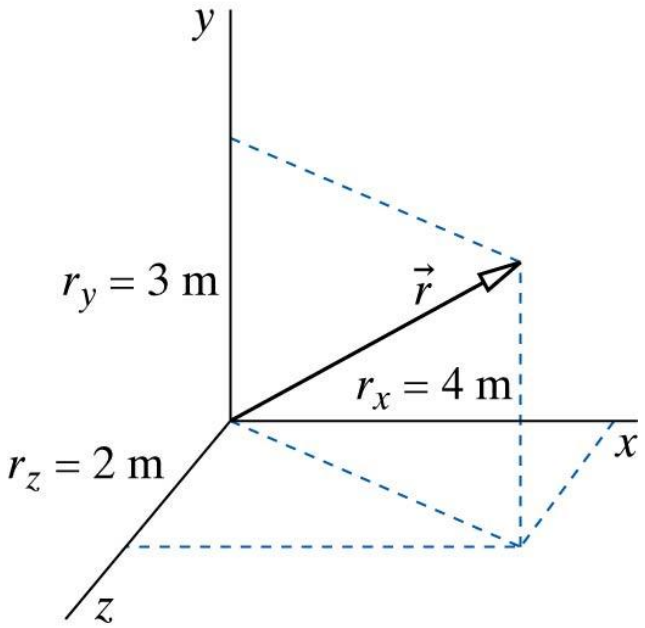


In general if $\vec{\mathbf{a}} = \langle a_x, a_y, a_z \rangle$ then

$$a = |\vec{\mathbf{a}}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

e.g. if $\vec{\mathbf{r}} = \langle 4, 3, 2 \rangle$ then

$$r = |\vec{\mathbf{r}}| = \sqrt{4^2 + 3^2 + 2^2} = 5.385 \text{ m}$$



Vectors: the magnitude

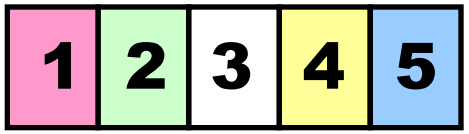
The magnitude of a vector is a scalar quantity.
e.g. speed is the magnitude of the vector velocity.

The magnitude of a vector is always positive.

A scalar p can also have a magnitude, $|p|$,

... which is p if $p \geq 0$,

... or $-p$ if $p < 0$.



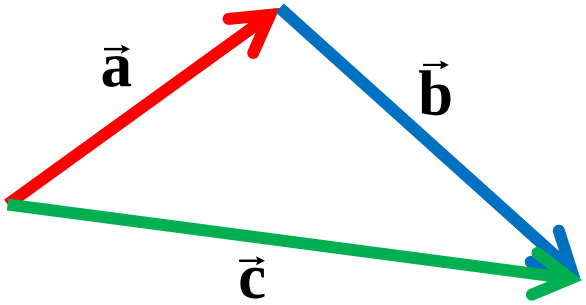
What is the magnitude of the vector $< 3, 5, -2 >$?

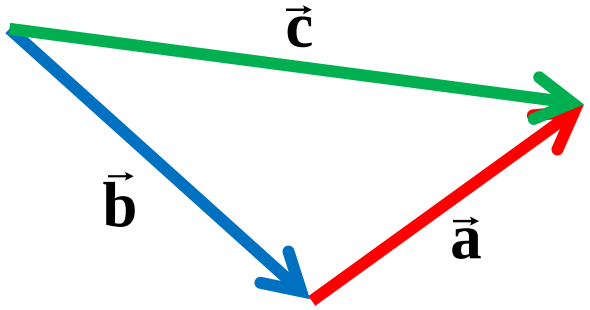
(1) 5.48
(2) 6.16
(3) 6.00
(4) 30.00
(5) 38.00

Vectors: addition

Vectors can be added geometrically, e.g. by means of a scale drawing.

$$\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$$





Vectors addition is commutative: $\vec{\mathbf{a}} + \vec{\mathbf{b}} = \vec{\mathbf{b}} + \vec{\mathbf{a}}$

Using components is usually better …

Vectors addition: component method

We can add vectors by adding their components.

Suppose $\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

Then: $\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

$$= \langle a_x, a_y, a_z \rangle + \langle b_x, b_y, b_z \rangle$$

$$= \langle a_x + b_x, a_y + b_y, a_z + b_z \rangle$$

$$= \langle c_x, c_y, c_z \rangle$$

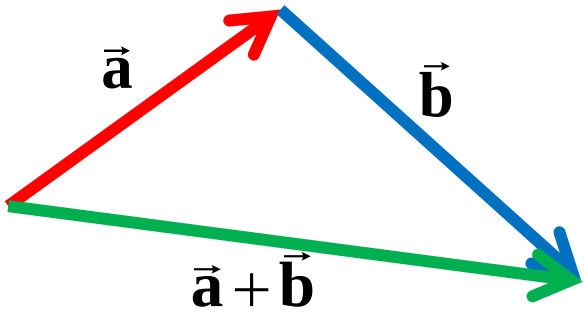
Vector addition: some properties

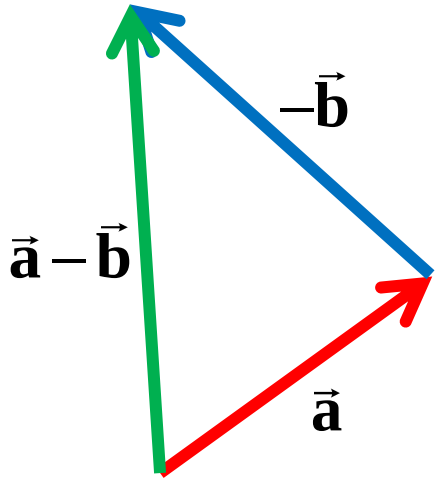
$\vec{a}+\vec{b}=\vec{b}+\vec{a}$ (commutative)

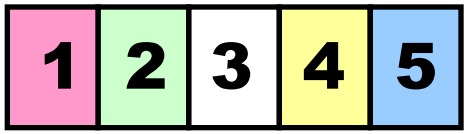
$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ (associative)

For each $\vec{b}$ there is a $-\vec{b}$ for which $\vec{b}+(-\vec{b})=0$

$\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$







What is $\langle 10, 20, -15 \rangle - \langle 5, -8, 7 \rangle$?

(1) 19
(2) 38.7
(3) $\langle 15, 12, 8 \rangle$
(4) $\langle 5, 28, -22 \rangle$
(5) $\langle 5, 12, -8 \rangle$

Vectors and scalars

A vector can be multiplied by a scalar ...

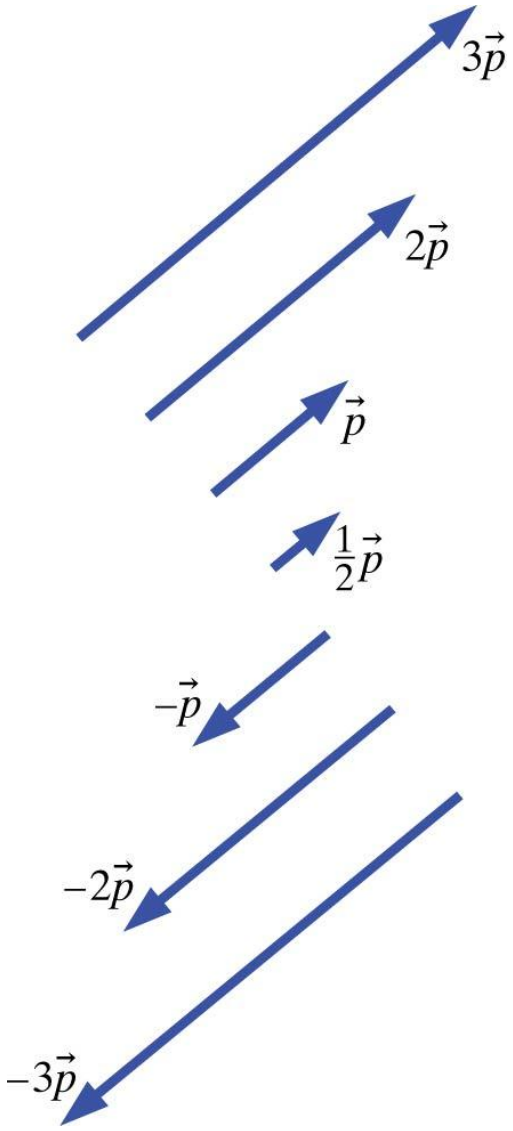
$\vec{\mathbf{a}} = q\vec{\mathbf{p}}$ means that $a_x = qp_x$

$a_y = qp_y$

$a_z = qp_z$

$$\vec{\mathbf{a}} = q\vec{\mathbf{p}} = q\langle p_x, p_y, p_z \rangle = \langle qp_x, qp_y, qp_z \rangle$$

The magnitude of $\vec{\mathbf{a}}$ is $a = |\vec{\mathbf{a}}| = |q||\vec{\mathbf{p}}|$



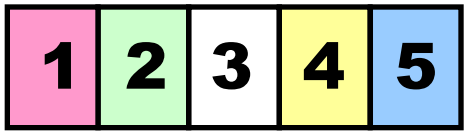
Unit vectors

Components are often useful when used with unit vector notation.

A unit vector is a vector with magnitude 1.
Only the direction is important.
The unit vector has no units.(!)

Notation: the unit vector in the direction $\vec{\mathbf{a}}$ is $\hat{\mathbf{a}}$.

It follows that $\hat{\mathbf{a}} = \dfrac{\vec{\mathbf{a}}}{a}$.



What is the unit vector in the direction of the vector $< 3, 5, -2 >$?

(1) $< 3, 5, -2 >$
(2) $< 1, 1, -1 >$
(3) $< 0.49, 0.81, 0.32 >$
(4) $< 0.49, 0.81, -0.32 >$
(5) $< 0.3, 0.5, -0.2 >$

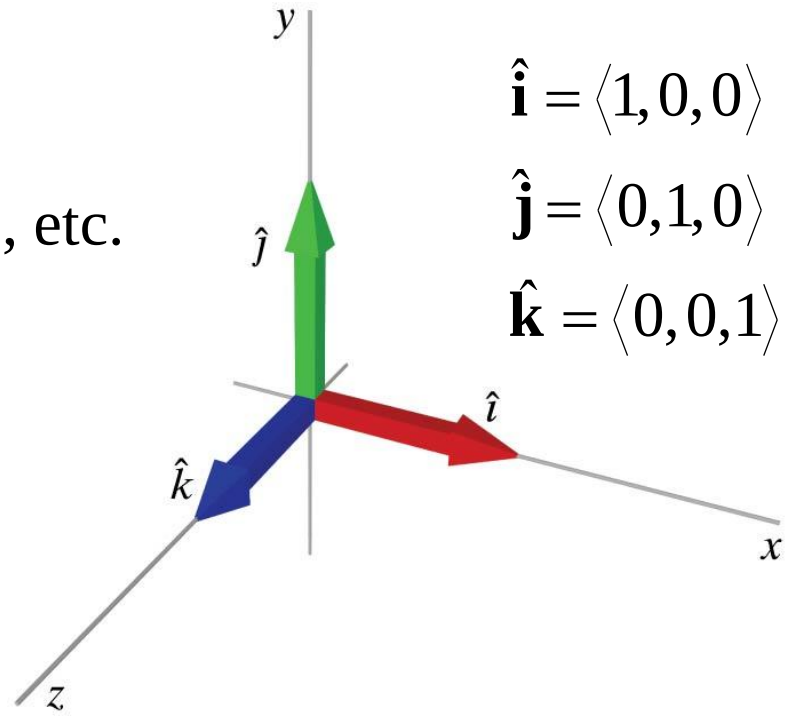
Unit vectors and component vectors

We denote the unit vectors along the *x*, *y*, *z* axes by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$.

Suppose $\vec{\mathbf{a}}$ has components a_x, a_y, a_z

Then $a_x\hat{\mathbf{i}}$ is a vector along the *x*-axis, etc.

Write $\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$



$\hat{\mathbf{i}} = \langle 1,0,0 \rangle$

$\hat{\mathbf{j}} = \langle 0,1,0 \rangle$

$\hat{\mathbf{k}} = \langle 0,0,1 \rangle$

Vector addition: unit vectors

Suppose $\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

Then: $\vec{\mathbf{a}} + \vec{\mathbf{b}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}} + b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}} + b_z\hat{\mathbf{k}}$

$= (a_x + b_x)\hat{\mathbf{i}} + (a_y + b_y)\hat{\mathbf{j}} + (a_z + b_z)\hat{\mathbf{k}}$

Vectors: summary

Vectors obey most of the usual rules of algebra:

Vectors can be added: $\vec{\mathbf{a}} + \vec{\mathbf{b}}$

Vectors can be multiplied by a scalar: $q\vec{\mathbf{a}}$

Then $\left|q\vec{\mathbf{a}}\right| = \left|q\right|\left|\vec{\mathbf{a}}\right|$

Then, e.g. $\vec{\mathbf{a}} - \vec{\mathbf{b}} = \vec{\mathbf{a}} + (-\vec{\mathbf{b}})$ so subtraction is well-defined.

Similarly, we can do: $p\left(\vec{\mathbf{a}} + \vec{\mathbf{b}}\right) = p\vec{\mathbf{a}} + p\vec{\mathbf{b}}$

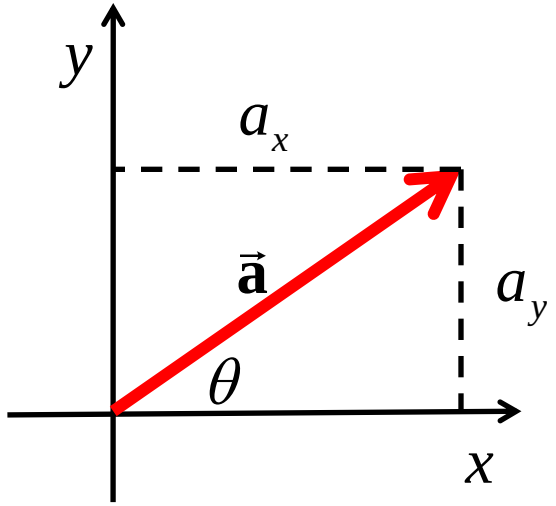
Vectors can be "multiplied together" … wait until later …

Vectors: what not to do …

Set a vector equal to a scalar.
Add a scalar to a vector.
Divide something by a vector.

Polar coordinates in 2D

The direction of $\vec{\mathbf{a}} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}}$ can be measured by the angle from the *x*-axis:



Then $a_x = a \cos\theta$

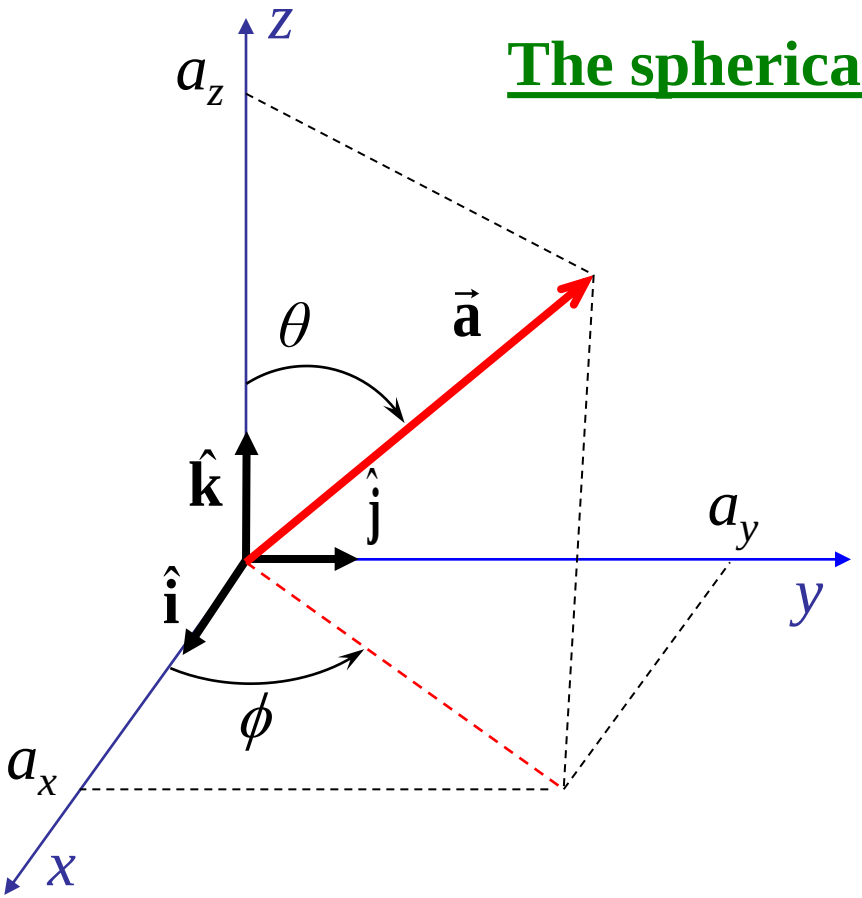
$a_y = a \sin\theta$

$$a = \sqrt{a_x^{\,2} + a_y^{\,2}}$$

$$\tan\theta = \frac{a_y}{a_x}$$

i.e. given components in one coordinate system, we can transform them to components in the other.

The spherical polar coordinate system in 3D



$$\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$$

$$a_x = a\ \cos\phi\ \sin\theta$$

$$a_y = a\ \sin\phi\ \sin\theta$$

$$a_z = a\ \cos\theta$$

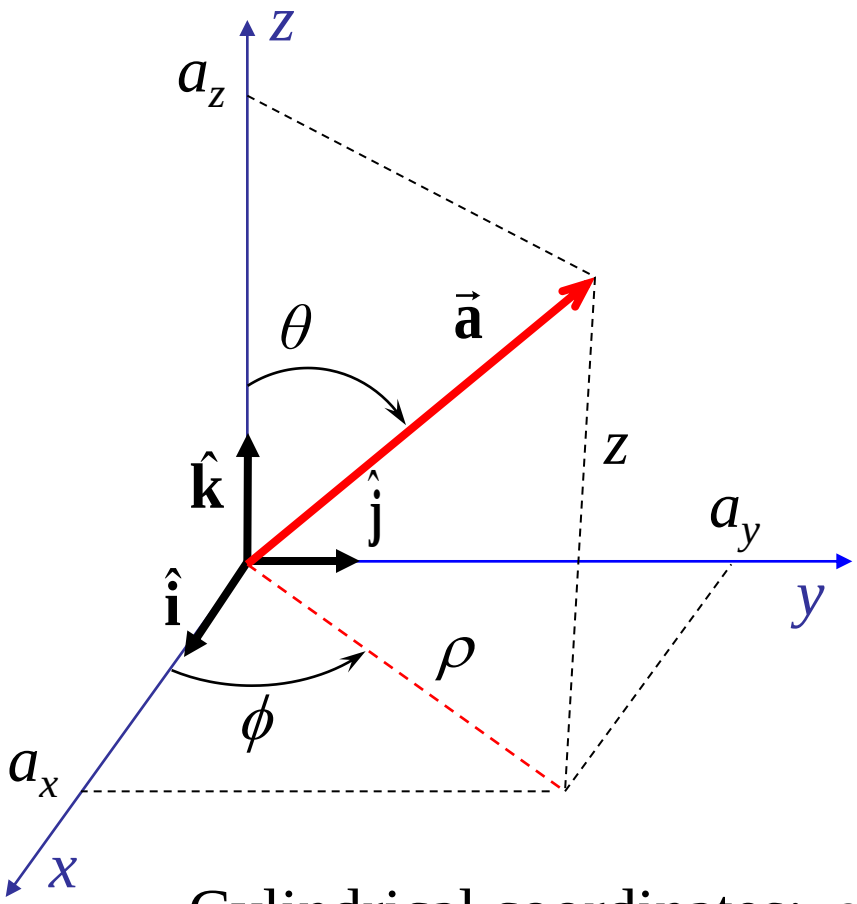
Spherical coordinates: a, θ, ϕ :

$$a = \sqrt{a_x^{\,2} + a_y^{\,2} + a_z^{\,2}}$$

$$\cos\theta = \frac{a_z}{a}$$

$$\tan\phi = \frac{a_y}{a_x}$$

The cylindrical polar coordinate system



$$\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$$

$$a_x = \rho\ \cos\phi$$

$$a_y = \rho\ \sin\phi$$

$$a_z = z$$

Cylindrical coordinates: ρ, θ, z :

$$\rho = \sqrt{a_x^{\,2} + a_y^{\,2}}$$

$$\tan\phi = \frac{a_y}{a_x}$$

$$z = a_z$$

Differentiation of vector functions

If $\vec{\mathbf{a}}(t) = a_x(t)\hat{\mathbf{i}} + a_y(t)\hat{\mathbf{j}} + a_z(t)\hat{\mathbf{k}}$

then $$\frac{d}{dt}\vec{\mathbf{a}}(t) = \frac{da_x(t)}{dt}\hat{\mathbf{i}} + \frac{da_y(t)}{dt}\hat{\mathbf{j}} + \frac{da_z(t)}{dt}\hat{\mathbf{k}}$$

Also: $$\frac{d}{dt}\left[\vec{\mathbf{a}}(t) + \vec{\mathbf{b}}(t)\right] = \frac{d\vec{\mathbf{a}}(t)}{dt} + \frac{d\vec{\mathbf{b}}(t)}{dt}$$

$$\frac{d}{dt}\left[c(t)\vec{\mathbf{a}}(t)\right] = \frac{dc(t)}{dt}\vec{\mathbf{a}}(t) + c(t)\frac{d\vec{\mathbf{a}}(t)}{dt}$$

Example of the time derivatives of a vector function

If position $\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$ m

then the instantaneous velocity $\vec{\mathbf{v}}(t) = \frac{d\vec{\mathbf{r}}(t)}{dt}$ m s^{-1}

$$\vec{\mathbf{v}}(t) = \frac{dx(t)}{dt}\hat{\mathbf{i}} + \frac{dy(t)}{dt}\hat{\mathbf{j}} + \frac{dz(t)}{dt}\hat{\mathbf{k}}$$

and the instantaneous acceleration $\vec{\mathbf{a}}(t) = \frac{d\vec{\mathbf{v}}(t)}{dt}$ m s^{-2}

$$\vec{\mathbf{a}}(t) = \frac{dv_x(t)}{dt}\hat{\mathbf{i}} + \frac{dv_y(t)}{dt}\hat{\mathbf{j}} + \frac{dv_z(t)}{dt}\hat{\mathbf{k}}$$

Units

Physics is based on measurement and experiment.
Units are required in order to express these measurements in a standard way.

A number referring to a physical quantity is meaningless without units.
Both sides of an equation must have the same units.
Use powers of 10 and prefixes to scale units.

Base units for **basic quantities** like distance, time and mass.
Derived units are convenient combinations of base units.

Base units

Length: Sizes, lengths, distances are measured in metres (abbreviated m). (L)

Time: Time intervals and durations are measured in seconds (abbreviated s). (T)

Mass: Masses are measured in kilograms (abbreviated kg). (M)

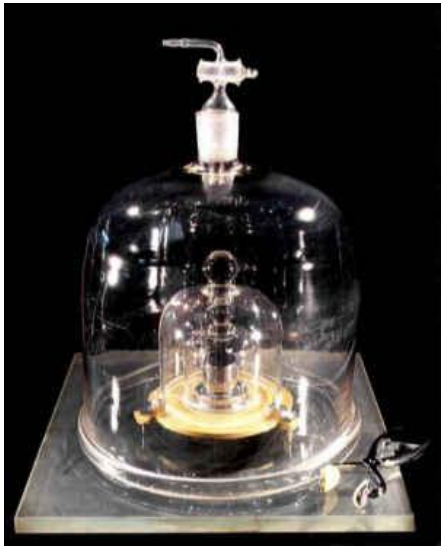
Also: ampere, kelvin, mole, candela.

Base units

The **metre** is defined as the distance light travels in vacuum in 1/299792458 seconds. Note that the speed of light is now, by definition, $c = 299792458 \text{ m s}^{-1}$.

The **second** is defined using the characteristic transition frequency of a particular kind of caesium atom in an atomic clock, as 9192631770 periods.

The **kilogram** is defined as the mass of a certain platinum-iridium alloy cylinder kept at the *International Bureau of Weights and Measures* in France.



http://physics.nist.gov/cuu/Units/

Derived units

Derived units are convenient combinations of the base units.
Often they are given their own names.
The combinations of units are termed the dimensions of the unit: the base units have dimensions of L, T and M.

The phrase [energy] can be read as “the dimensions of energy”.

For example, we have $[\text{energy}] = \dfrac{[\text{mass}]\,[\text{length}]^2}{[\text{time}]^2} = \dfrac{\text{M L}^2}{\text{T}^2}$

The unit of energy is the joule, abbreviated J.

Note that we can treat the combination of units algebraically.

Units: SI prefixes

Powers of ten

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

Newtonian mechanics

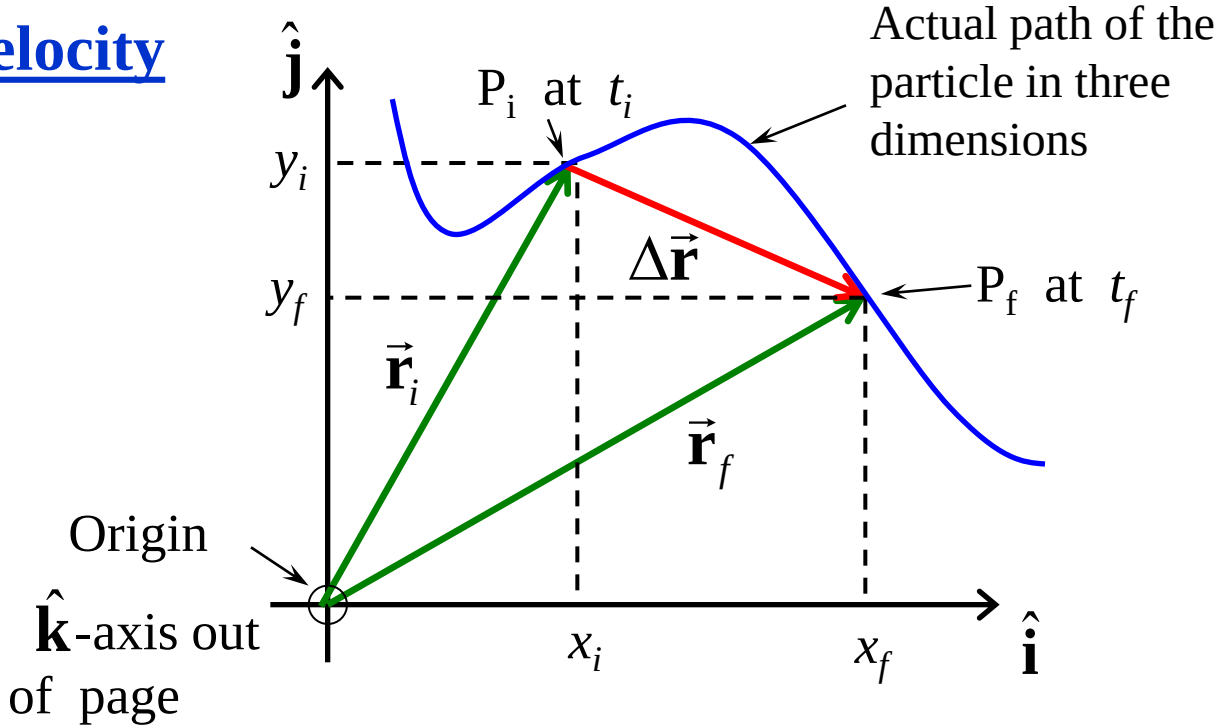
The "flavour" of mechanics that we will use is called "Newtonian mechanics" … developed by Sir Isaac Newton …

… which has a number of conditions which are important …

… Nature is assumed to be **continuous** (which was shown not to be quite true by Einstein's Special Relativity, and quantum mechanics.)

… the equations of motion are applicable only to **point particles** (in most of the situations we will consider in this course).

Position and velocity

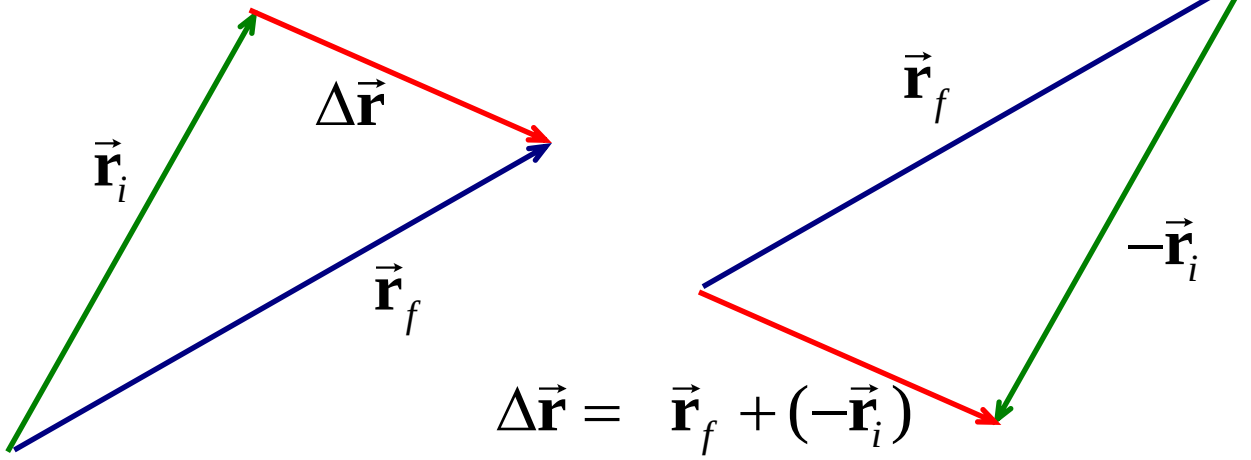


Consider a particle that follows the curved (blue) path in space.
At time t_i it is at position P_i and time t_f it is at position P_f.
To describe the motion of the particle, we use a three dimensional Cartesian coordinate system as shown.
Therefore at $t = t_i$, the particle is at position $\vec{\mathbf{r}}_i = x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}$
and $t = t_f$, the particle is at position $\vec{\mathbf{r}}_f = x_f\hat{\mathbf{i}} + y_f\hat{\mathbf{j}} + z_f\hat{\mathbf{k}}$

The **displacement vector** is

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i = \left(x_f - x_i\right)\hat{\mathbf{i}} + \left(y_f - y_i\right)\hat{\mathbf{j}} + \left(z_f - z_i\right)\hat{\mathbf{k}}$$

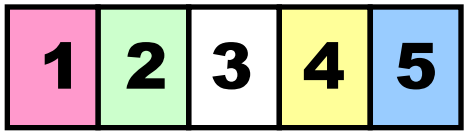
$$= \vec{\mathbf{r}}_f + (-\vec{\mathbf{r}}_i)$$



Of course $\left|\Delta\vec{\mathbf{r}}\right|$ is <u>not</u> necessarily the distance from P_i to P_f.

In general an **instantaneous position vector** $\vec{\mathbf{r}}(t)$ describes the position of a particle at a particular instant in time relative to the origin of a set of coordinate axes:

$$\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$$



A proton is at location $< 0, 3, -2 >$ m.
An electron is at location $< -1, 0, -6 >$ m.
What is the relative position vector from the proton to the electron?

(1) $< -1, 3, -8 >$ m
(2) $< -1, -3, -4 >$ m
(3) $< 1, 3, 4 >$ m
(4) $< 1, -3, 8 >$ m
(5) $< 1, 0, 6 >$ m

The **average velocity** vector $\vec{\mathbf{v}}_{av} = \dfrac{\Delta\vec{\mathbf{r}}}{\Delta t} = \dfrac{\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i}{t_f - t_i}$

The **instantaneous velocity** vector $\vec{\mathbf{v}}(t) = \dfrac{d\vec{\mathbf{r}}(t)}{dt}$

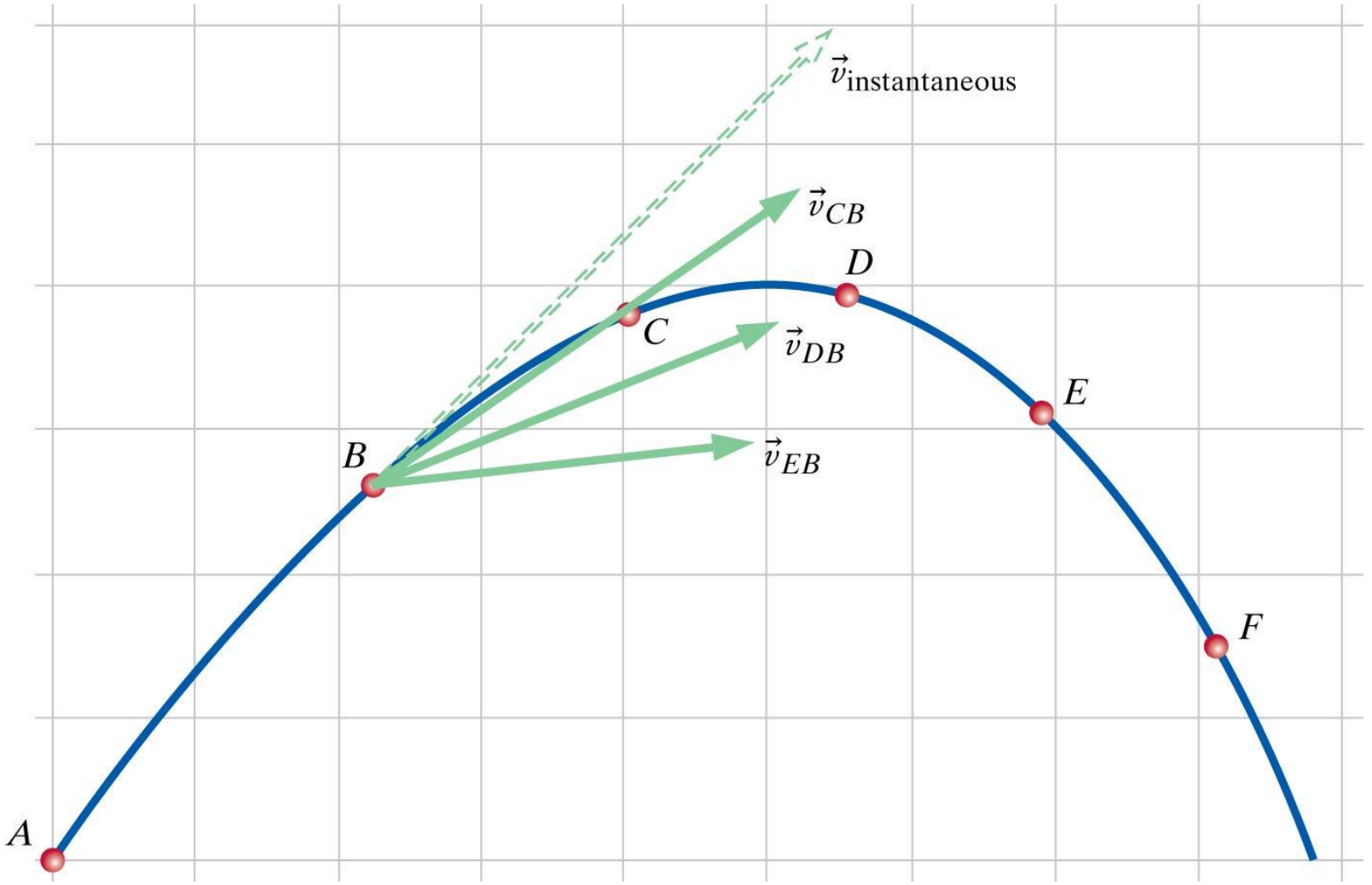
$$= \frac{dx(t)}{dt}\hat{\mathbf{i}} + \frac{dy(t)}{dt}\hat{\mathbf{j}} + \frac{dz(t)}{dt}\hat{\mathbf{k}}$$

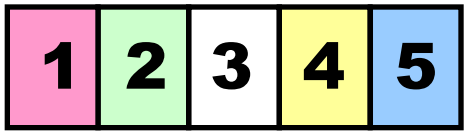
The **average acceleration** vector $\vec{\mathbf{a}}_{av} = \dfrac{\Delta\vec{\mathbf{v}}}{\Delta t} = \dfrac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i}$

The **instantaneous acceleration** vector $\vec{\mathbf{a}}(t) = \dfrac{d\vec{\mathbf{v}}(t)}{dt}$

$$= \frac{d^2x(t)}{dt^2}\hat{\mathbf{i}} + \frac{d^2y(t)}{dt^2}\hat{\mathbf{j}} + \frac{d^2z(t)}{dt^2}\hat{\mathbf{k}}$$

Instantaneous and average velocity





A bee flies in a straight line at constant speed.
At 15 s after 9 a.m., the bee's position is $< 2, 4, 0 >$ m.
At 15.5 s after 9 a.m., the bee's position is $< 3, 3.5, 0 >$ m.

What is the **average velocity** of the bee?

(1) $< 6, 7, 0 >$ m s^{-1}
(2) $< 0.193, 0.225, 0 >$ m s^{-1}
(3) 2.236 m s^{-1}
(4) $< 0.500, -0.250, 0 >$ m s^{-1}
(5) $< 2.000, -1.000, 0 >$ m s^{-1}

Predicting a new position

Rewrite the average velocity formula as

$$\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i = \vec{\mathbf{v}}_{av}\Delta t$$

From this we obtain an update formula for the position

$$\begin{aligned}\vec{\mathbf{r}}_f &= \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_{av}\Delta t \\ &= \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_{av}\left(t_f - t_i\right)\end{aligned}$$

We will use this throughout the course.

Note that this formula is exact
(but calculating $\vec{\mathbf{v}}_{av}$ may be difficult).

The update formula

The update formula $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_{av}\left(t_f - t_i\right)$ is a vector equation.

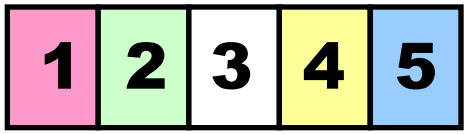
We can write it as three equations in the coordinates

$$x_f = x_i + v_{av,x}\left(t_f - t_i\right)$$

$$y_f = y_i + v_{av,y}\left(t_f - t_i\right)$$

$$z_f = z_i + v_{av,z}\left(t_f - t_i\right)$$

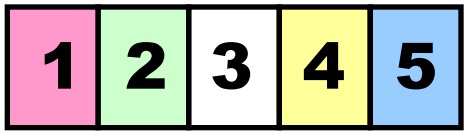
If $\vec{\mathbf{v}}_{av}$ does not change in time, these are the equations for a straight line.



At 15 s after 10 a.m. two bees are observed to be at position < 2, 4, 0> m.
Bee #1 flies in a straight line with constant speed and arrives at position < 3, 3.5, 0 > m at 15.5 s after 10 a.m.
Bee #2 buzzes around, repeatedly changing speed and direction, sometimes going quickly and other times just hovering in the air, but it also arrives at position < 3, 3.5, 0 > m at 15.5 s after 10 a.m.

Which statement about their average velocities is correct?

(1) The magnitude of Bee #1's average velocity is greater.
(2) The magnitude of Bee #2's average velocity is greater.
(3) The two bees have the same velocity at all times.
(4) The two bees have the same average velocity although their velocity at a given time may not be the same.



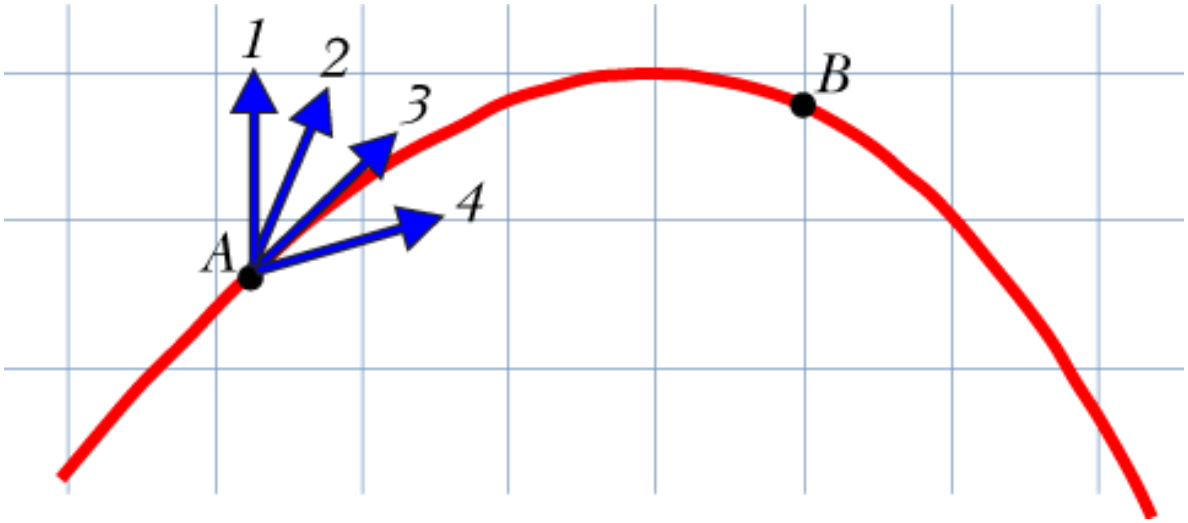
At 12.18 s after 1:30 p.m., a ball's position is $< 20, 8, -12 >$ m, and the ball's velocity is $< 9, -4, 6 >$ m s^{-1}.

What is the (vector) position of the ball at 12.21 s after 1:30 p.m.? Assume that the ball's velocity does not change significantly in this short time interval.

(1) 24.75 m
(2) $< 20.27, 7.88, -11.82 >$ m
(3) $< 0.27, -0.12, 0.18 >$ m
(4) $< 129.62, -40.72, 61.08 >$ m
(5) $< 129.89, -40.84, 61.26 >$ m

1	2	3	4	5

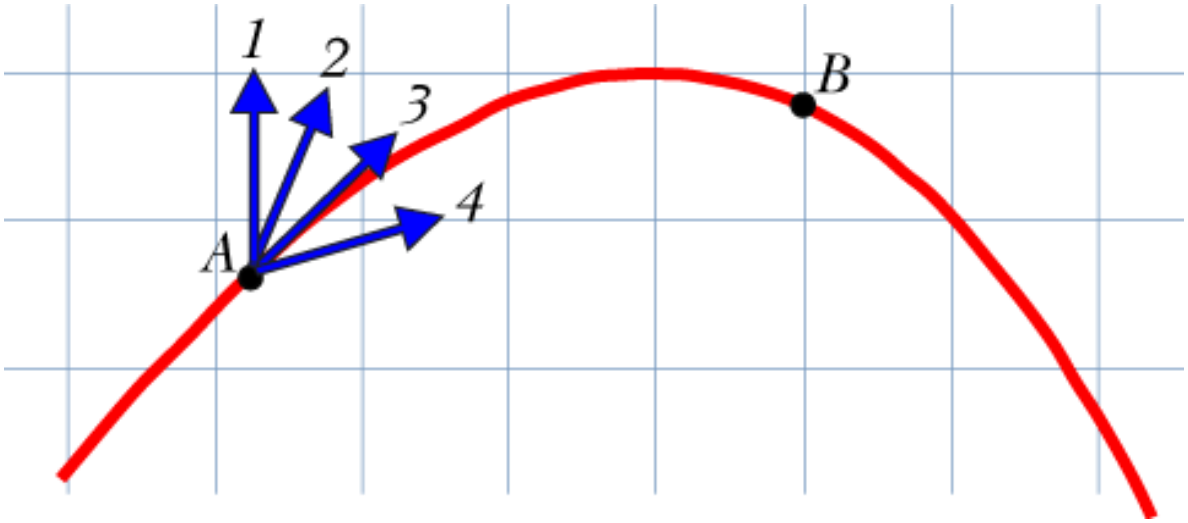
A ball travels through the air. Part of its trajectory is shown in red.



Which arrow best represents the direction of the **average velocity** of the ball as it travels from location *A* to location *B*?

1	2	3	4	5

A ball travels through the air. Part of its trajectory is shown in red.



Which arrow best represents the direction of the **instantaneous velocity** of the ball when it passes through location *A*?

Example

The position of a 3 kg particle as a function of time is given by:

$$\vec{\mathbf{r}}(t) = (5t^2 - 2t)\hat{\mathbf{i}} + (3t + 8)\hat{\mathbf{j}} \quad \text{metres}$$

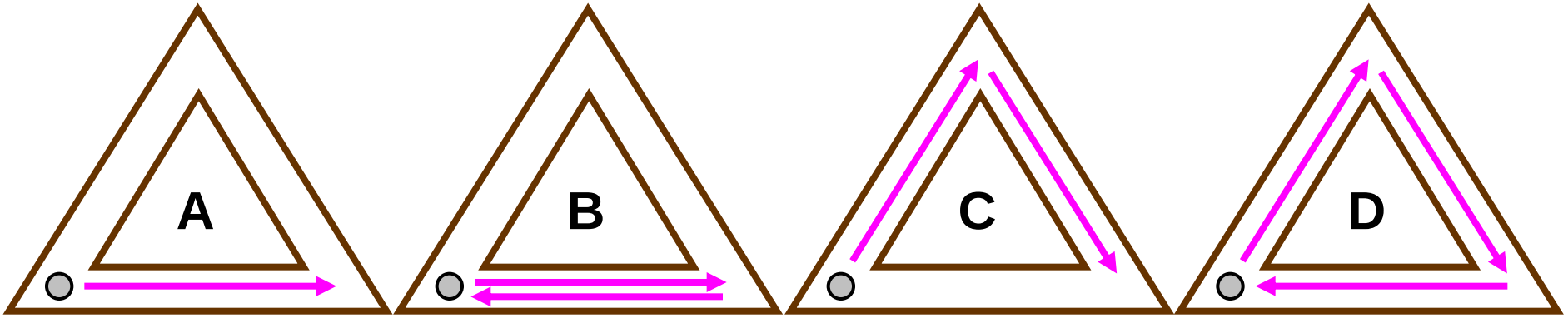
(a) What is the position of the object at $t = 2$ s?
(b) Write down an expression for the velocity of the object as a function of time.
(c) What is the velocity of the object at $t = 2$ s?
(d) What is the displacement of the object between $t = 0$ s and $t = 2$ s ?
(e) What is the average velocity of the object between $t = 0$ s and $t = 2$ s?

Example

A particle moves 3 metres in the $\hat{\mathbf{i}}$-direction for 6 seconds, and then 6 metres in a direction 30° to the $\hat{\mathbf{i}}$-direction for another 6 seconds. What is the average velocity of the particle?

Example

Four different mice (labeled A, B, C and D) ran the triangular maze shown below. They started in the lower left hand corner and followed the paths of the arrows. The times they took are shown below each figure.



For each item below, write down the letters of all the mice that fit the description.

(a) This mouse had the greatest average speed.
(b) This mouse had the greatest total displacement.
(c) This mouse had an average velocity that points in this direction →
(d) This mouse had the greatest average velocity.

Momentum

Newton's first law of motion,

An object moves in a straight line and at constant speed except to the extent that it interacts with other objects.

is often known as the "law of inertia".

Inertia is the resistance of a body to changes in motion.
We want to relate a change in velocity to the strength of interaction.

What quantity best represents this inertial aspect?
Both mass and velocity are important to consider.

Momentum

Momentum is a quantity that involves a product of mass and velocity.
Momentum is denoted by the symbol $\vec{\mathbf{p}}$.

We might expect $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

However, experiment and a deeper understanding of the nature of space and time has shown that the momentum is defined by:

$$\vec{\mathbf{p}} = \gamma m\vec{\mathbf{v}}$$

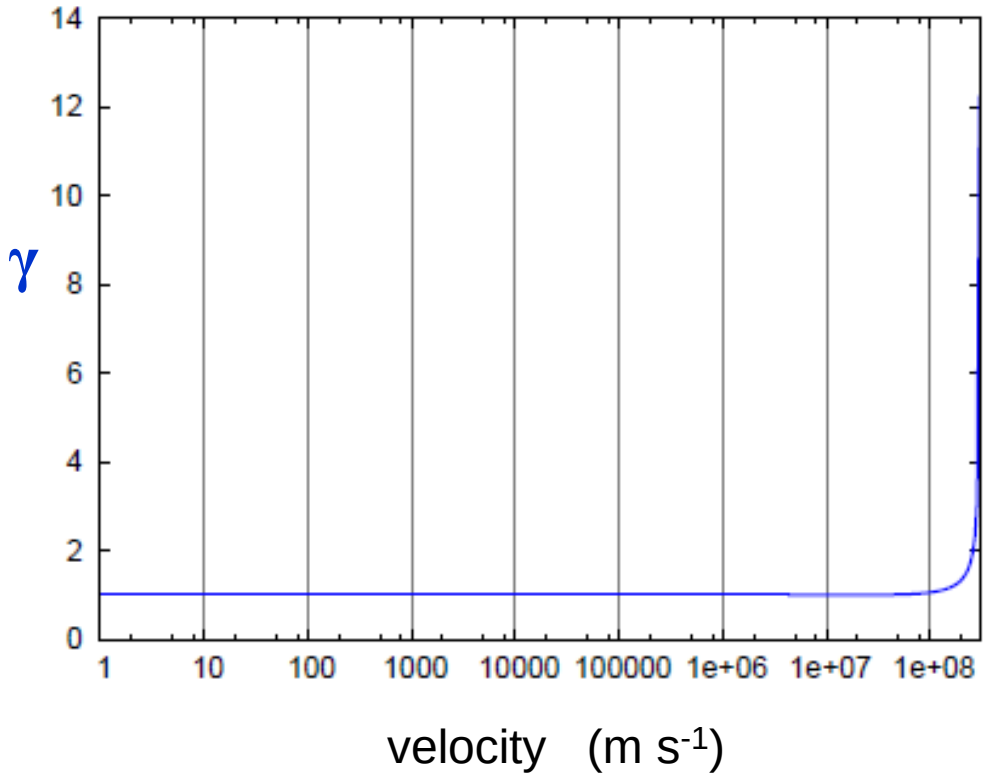
where the Lorentz factor is $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

and c is the speed of light (3×10^8 m s^{-1}).

The Lorentz factor γ

For many everyday speeds, γ is very close to 1.

e.g. for a speed of 3000000 m s^{-1}, γ = 1.00005.



So we can say that if $v \ll c$, say, $v \sim 0.01c$, then

$$\vec{\mathbf{p}} = 1.000 m\vec{\mathbf{v}} = m\vec{\mathbf{v}}$$

Whether we accept this depends on how accurately we need the value; often the approximation is acceptable.

Momentum

The momentum is a vector in the same direction as the velocity.

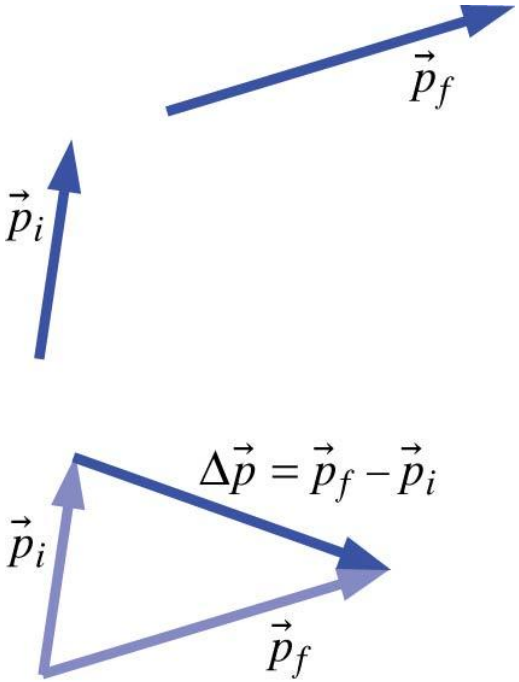
Note that it is not, in general, a constant multiple of the velocity.

Units of momentum: kg m s^{-1}.

We will see that the momentum has important properties that we can exploit in analysing motion, especially if collisions are involved.

(Conservation of momentum: to come).
This makes it much more fundamental and useful than velocity.

Change in momentum



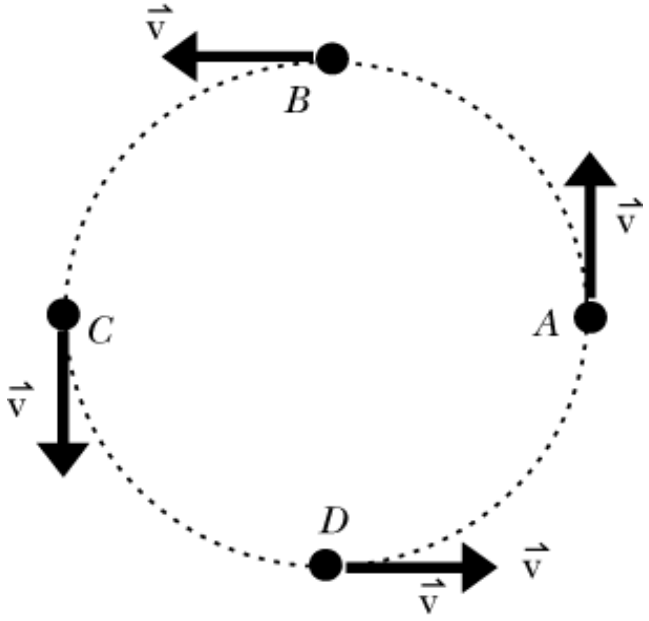
Simple form of Newton 1:
Momentum of a particle is constant in the absence of interactions.

So the change in momentum is related to the presence of an interaction.

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$$

The **average rate of change in momentum**: $\dfrac{\Delta\vec{\mathbf{p}}}{\Delta t} = \dfrac{\vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i}{t_f - t_i}$

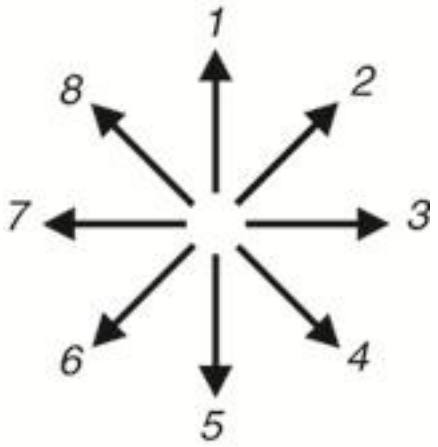
The **instantaneous rate of change in momentum**: $\dfrac{d\vec{\mathbf{p}}}{dt} = \lim_{\Delta t \to 0} \dfrac{\Delta\vec{\mathbf{p}}}{\Delta t}$



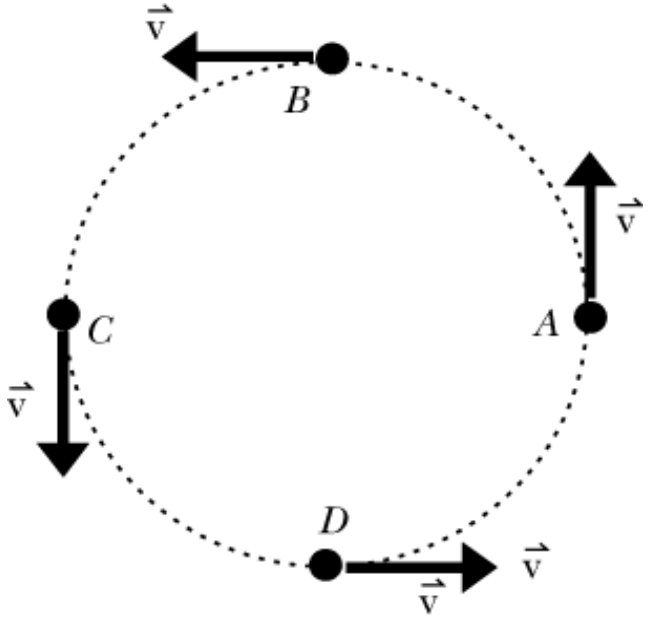
1	2	3	4	5

A child rides on a merry-go-round, traveling from location *A* to location *C* at a constant speed.

What is the direction of $\Delta\vec{p}$, the change in the child's momentum, between locations *A* and *C* ?



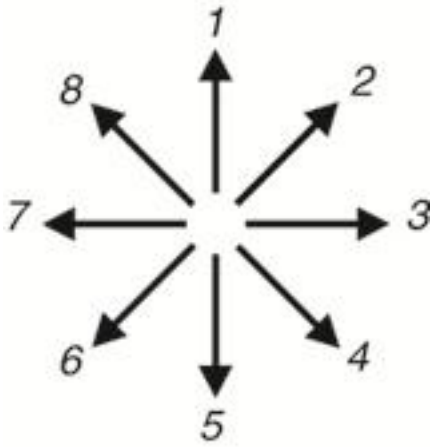
9 zero magnitude



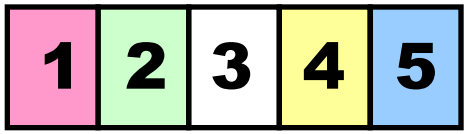
1	2	3	4	5

A child rides on a merry-go-round, traveling from location *A* to location *C* at a constant speed.

What is the direction of $\Delta\vec{p}$, the change in the child's momentum, between locations *A* and *B* ?



9 zero magnitude



Suppose you are driving a 1000 kg car at 20 m s^{-1} in the +*x* direction. After making a 180 degree turn, you drive the car at 20 m s^{-1} in the –*x* (opposite) direction. What is the magnitude of the change of the momentum of the car $|\Delta\vec{p}|$?

(1) 0 kg m s^{-1}
(2) 2.0e4 kg m s^{-1}
(3) 4.0e4 kg m s^{-1}
(4) 6.0e4 kg m s^{-1}
(5) 8.0e4 kg m s^{-1}

Principle of relativity

Galileo: laws of mechanics are the same for observers in uniform motion as for observers at rest.

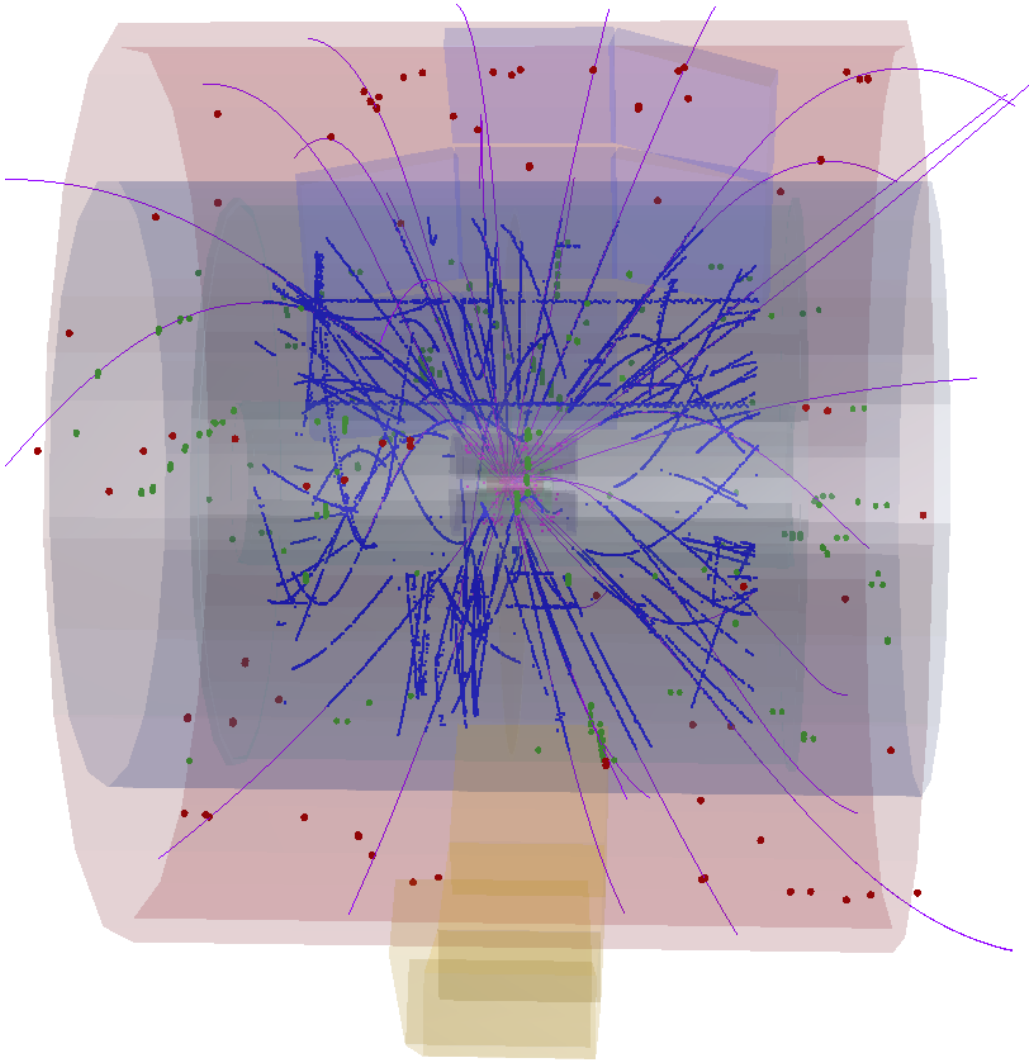
Einstein: laws of physics are the same for observers in uniform motion as for observers at rest.

Equivalent to Newton's First Law.

M&I

Chapter 2

The Momentum Principle



System and surroundings

In order to apply the momentum principle we must separate the world into the system whose momentum change we will calculate, and the surroundings which exert the forces on the system.

Only forces that are exerted across the boundary (‘external forces’) are important.

Internal forces cancel out in pairs.

The momentum principle

Newton's first law suggests that "the stronger the interaction, the bigger the change in momentum".

Experience also suggests that "the longer the interaction, the bigger the change in momentum".

The quantitative form is expressed as the **momentum principle** (or Newton's second law):

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{net}\Delta t \qquad \text{for small } \Delta t$$

Change of momentum is equal to the net force acting on an object times the duration of the interaction.

Force

We quantify interaction by the concept of force.

A force has a magnitude and is exerted in a certain direction.

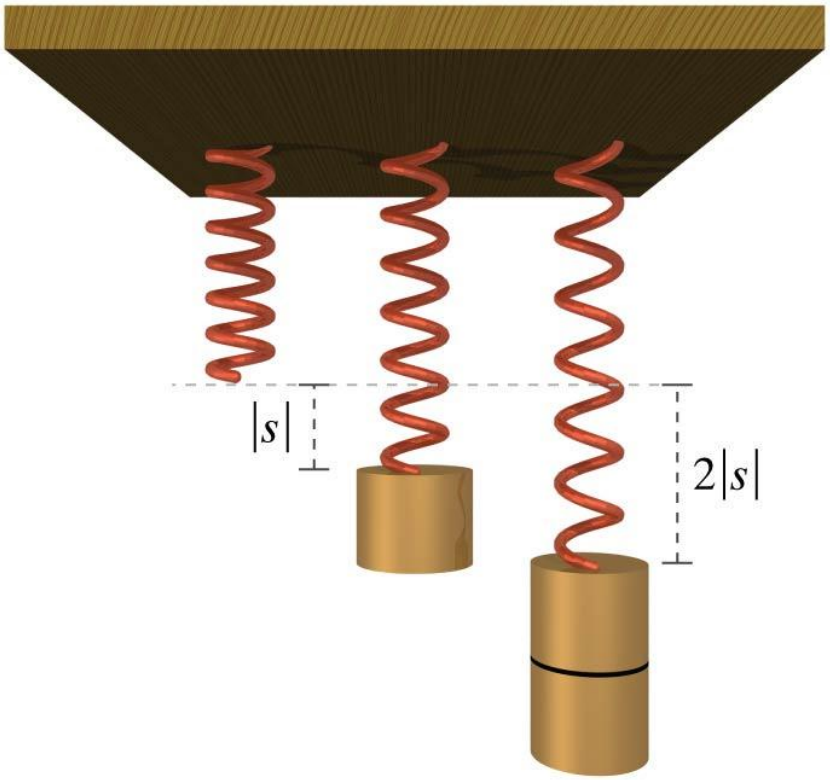
It is described by a vector $\vec{\mathbf{F}}$.

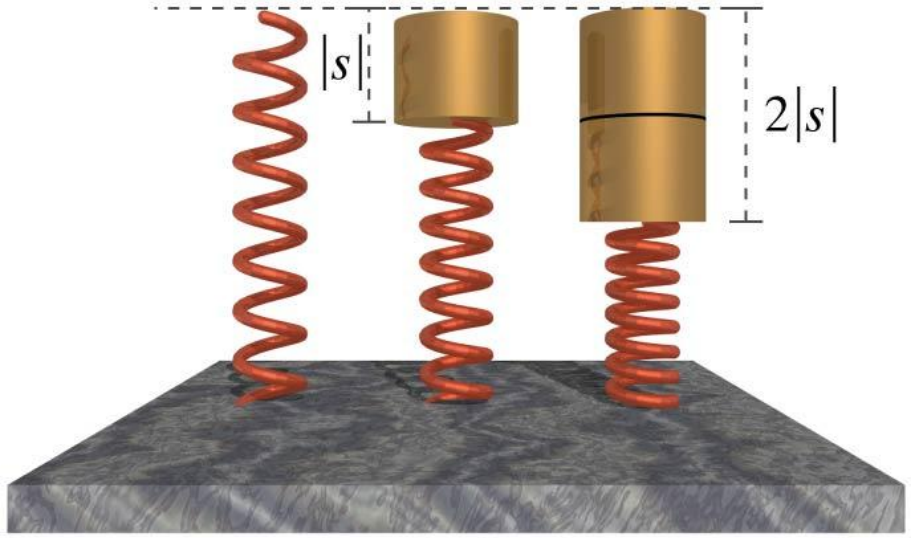
Units: kg m s^{-2}, also known as the **newton**, N.

Examples of force:

Force of gravity between Earth and Sun.
Electrostatic force between two protons.
Weight.
Force exerted by a spring.

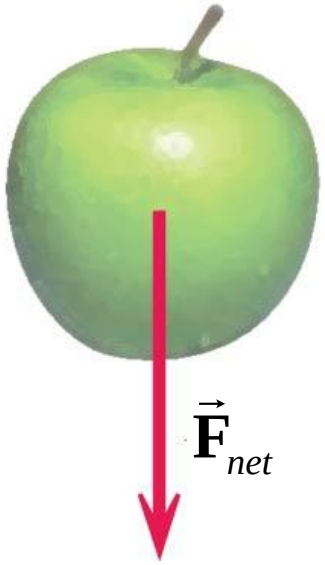
How do we measure a force?


|s|
2|s|


|s|
2|s|

Definition of net force

The net force on an object is the vector sum of the individual forces exerted on it by all other objects.



$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \ldots = \sum_i \vec{\mathbf{F}}_i$$

Impulse

The product of force and a time interval is called **impulse**.

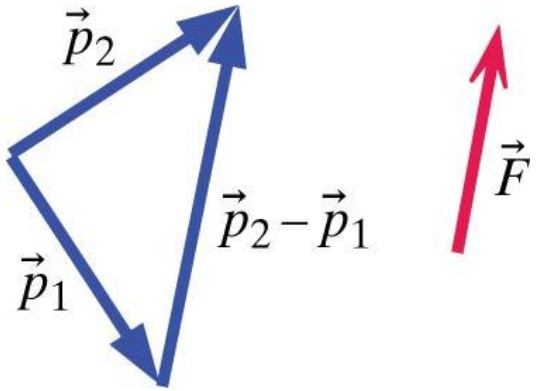
$$\textbf{impulse} \equiv \vec{\mathbf{F}}\Delta t$$

Units: Ns.

The momentum principle:

The change of momentum of an object is equal to the net impulse applied to it.

i.e. $\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \vec{\mathbf{F}}_{net}\Delta t$



The momentum update formula

The Momentum Principle can be written as an update formula:

$$\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i + \vec{\mathbf{F}}_{net}\Delta t$$

for small Δt … why?

Remember that this is three scalar equations:

$$p_{fx} = p_{ix} + F_{net,x}\Delta t$$
$$p_{fy} = p_{iy} + F_{net,y}\Delta t$$
$$p_{fz} = p_{iz} + F_{net,z}\Delta t$$

How do we use this?

Applying the momentum principle

To analyse the motion of a real world system. . .

1. Separate the world into system and surroundings

2. Determine which forces you need consider
... make a list, draw a diagram

3. Choose the time interval

4. Apply the momentum principle

5. Check

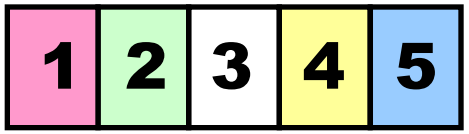
Relativistic updates

For fast moving particles ($v \approx c$) the momentum is

$$\vec{\mathbf{p}} = \gamma m \vec{\mathbf{v}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m\vec{\mathbf{v}}$$

For the position update formula, need $\vec{\mathbf{v}}$ in terms of $\vec{\mathbf{p}}$.
The formula is "easily" inverted:

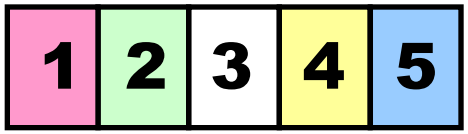
$$\vec{\mathbf{v}} = \frac{\vec{\mathbf{p}}/m}{\sqrt{1 + \left(\frac{|\vec{\mathbf{p}}|}{mc} \right)^2}}$$



An object is moving in the +*x* direction.
Which of the following statements about the net force acting on the object could be true?

A. The net force is in the +*x* direction
B. The net force is in the –*x* direction
C. The net force is zero

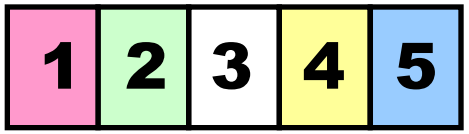
(1) A only
(2) B only
(3) C only
(4) A and B
(5) B and C
(6) A and C
(7) A, B, and C



Cart A moves to the left at nearly constant speed.
Cart B moves to the left, gradually speeding up.
Cart C moves to the left, gradually slowing down.

Which cart(s) experience a net force to the left?

(1) A only
(2) B only
(3) C only
(4) A and B
(5) B and C
(6) A and C
(7) A, B, and C



The *x*-component of momentum of an object is found to increase with time:

$t = 0$ s $\quad p_x = 30$ kg m/s

$t = 1$ s $\quad p_x = 40$ kg m/s

$t = 2$ s $\quad p_x = 50$ kg m/s

$t = 3$ s $\quad p_x = 60$ kg m/s

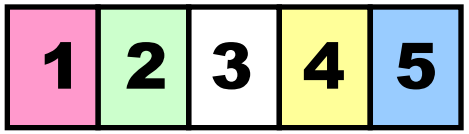
What can you conclude about the *x*-component of the net force acting on the object?

(1) $F_{net,x} = 0$

(2) $F_{net,x}$ is constant

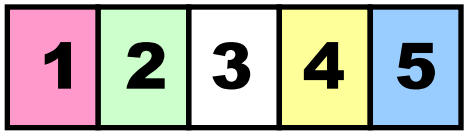
(3) $F_{net,x}$ is increasing with time

(4) Not enough information is given to determine which is true.



A hockey puck is sliding along the ice with nearly constant momentum < 10, 0, 5 > kg m/s when it is suddenly struck by a hockey stick with a force < 0, 0, 2000 > N that lasts for only 3 milliseconds (3e-3 s).
What is the new (vector) momentum of the puck?

(1) < 10, 0, 11 > kg m/s
(2) < 0, 0, 6 > kg m/s
(3) 14.86 kg m/s
(4) < 16, 0, 11 > kg m/s
(5) < 0, 0, 30 > kg m/s



You push a book across a table. In order to keep the book moving with constant momentum, you have to keep pushing with a constant force.
Which statement explains this?

(1) A net force is necessary to keep an object moving.
(2) To make the net force on the book zero, you must push with a force equal and opposite to the friction force on the book.
(3) The force you exert must be slightly larger than the friction force.

1	2	3	4	5

Inside a spaceship in outer space there is a small steel ball.
At a particular instant, the ball has momentum
< –8, 3, 0 > kg m/s and is pulled by a string, which exerts
a force < 20, –10, 0 > N on the ball.
What is the ball's (vector) momentum 2 seconds later?

(1) < –28, 23, 0 > kg m/s
(2) < 12, –7, 0 > kg m/s
(3) 36.2 kg m/s
(4) < 32, –17, 0 > kg m/s
(5) < 40, –20, 0 > kg m/s

Prediction and the momentum principle

We now have a way to predict the future of a system:

1. Identify the system and surroundings ("rest of the universe") ... identify the interactions.

2. Identify the initial conditions.

3. Use the momentum principle to update the momentum.

4. Use the position update formula to update the position.

5. Solve for any remaining unknowns.

Updating position when momentum is changing

We need to find the average velocity $\vec{\mathbf{v}}_{av}$ in order to use the update equation for position.

Can't use: $\vec{\mathbf{v}}_{av} = \dfrac{\Delta\vec{\mathbf{r}}}{\Delta t} = \dfrac{\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i}{\Delta t}$

A good approximation is $\vec{\mathbf{v}}_{av} = \dfrac{\vec{\mathbf{v}}_i + \vec{\mathbf{v}}_f}{2}$

This is exact if the rate at which the velocity is changing is constant.

Momentum change with changing force

In real world situations it is common for both the magnitude and direction of forces to change.

If the net force on system is not constant over a time interval, then divide up the time interval into several smaller intervals, until the net force is approximately constant over each interval.

Then the total change in momentum is

$$\Delta\vec{\mathbf{p}} = \Delta\vec{\mathbf{p}}_1 + \Delta\vec{\mathbf{p}}_2 + \Delta\vec{\mathbf{p}}_3 + \ldots$$

where

$$\Delta\vec{\mathbf{p}}_1 = \vec{\mathbf{p}}_1 - \Delta\vec{\mathbf{p}}_0$$
$$\Delta\vec{\mathbf{p}}_2 = \vec{\mathbf{p}}_2 - \Delta\vec{\mathbf{p}}_1$$
$$\Delta\vec{\mathbf{p}}_3 = \vec{\mathbf{p}}_3 - \Delta\vec{\mathbf{p}}_2$$

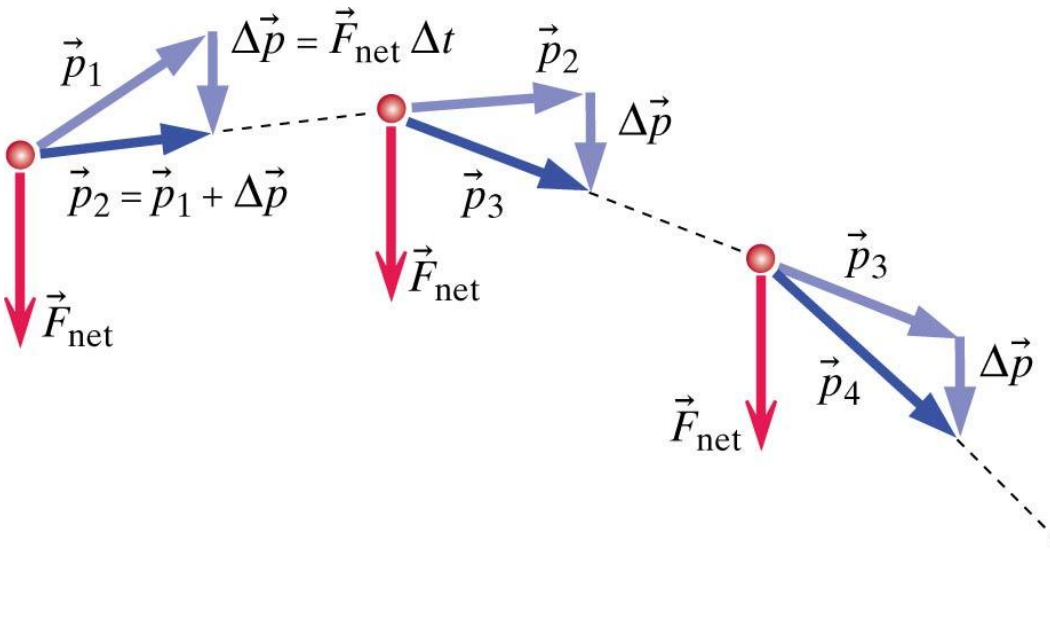
Iterative prediction of motion

Calculate the (vector) forces acting on the system.

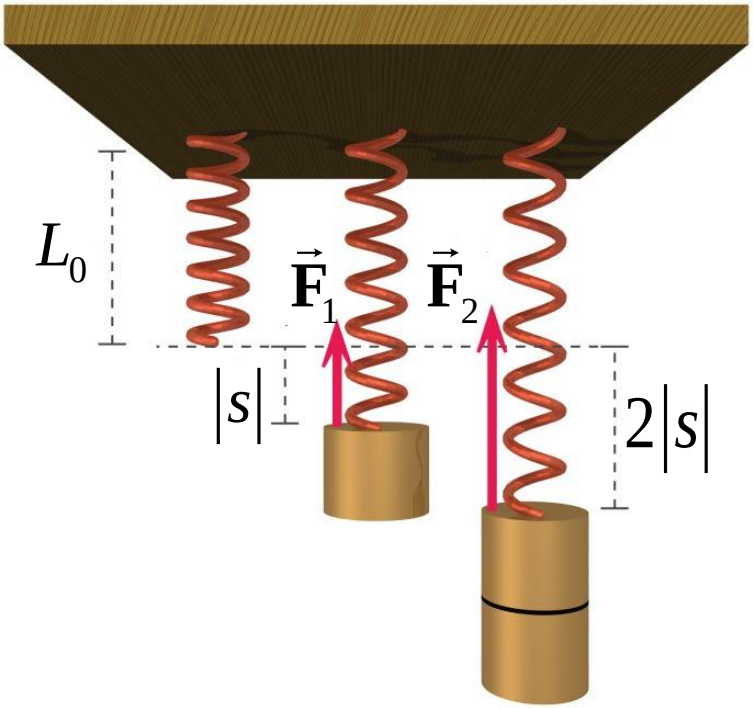
Update the momentum of the system: $\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i + \vec{\mathbf{F}}_{net}\Delta t$

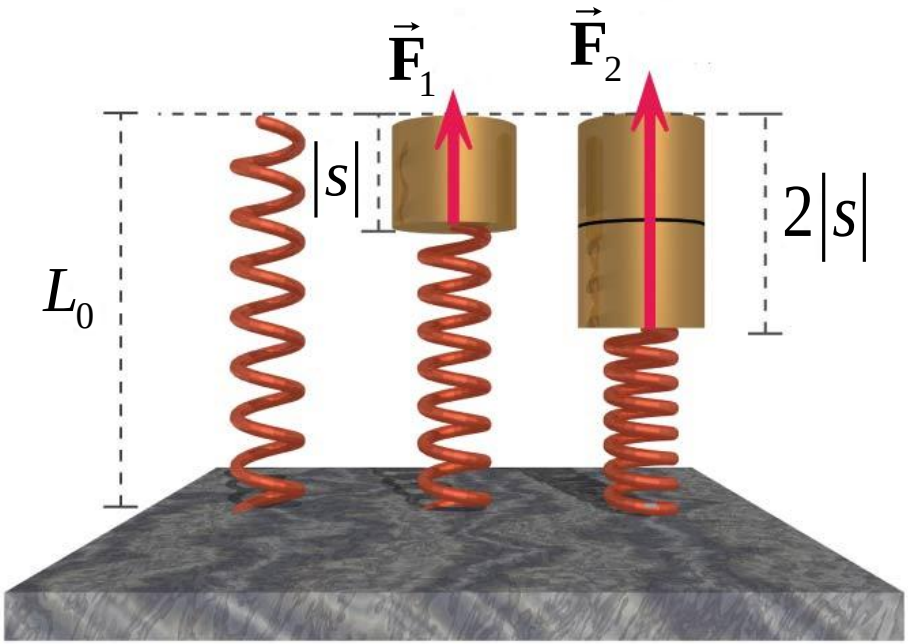
Update the position: $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_{av}\Delta t$

Repeat



The spring force



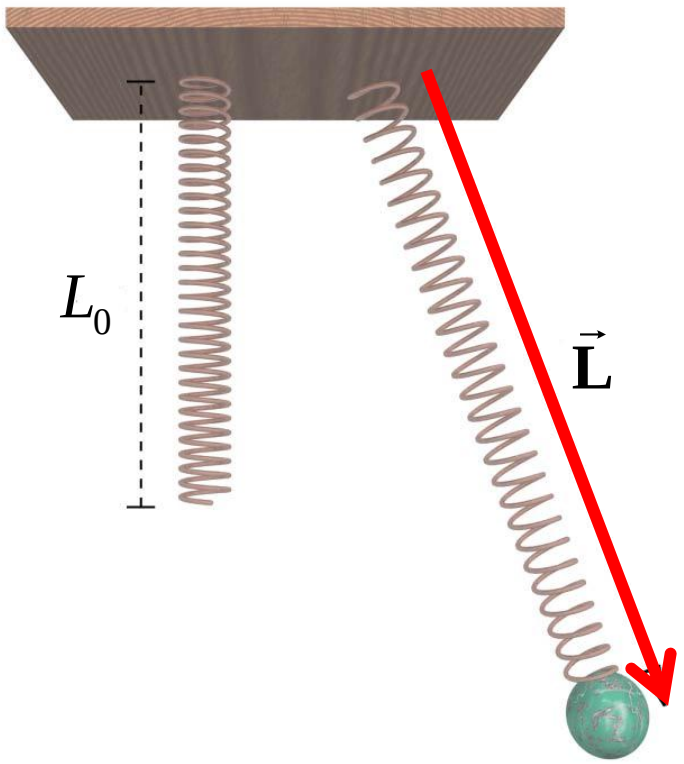


Hooke's law

$$\left|\vec{\mathbf{F}}_{spring}\right| = k_s\,|s| \qquad \text{where} \quad s = L - L_0$$

and k_s is the "spring constant" Units: N m^{-1}

The vector spring force



Relative position vector $\vec{\mathbf{L}}$

$$\vec{\mathbf{L}} = \left|\vec{\mathbf{L}}\right|\hat{\mathbf{L}}$$

then $$s = \left|\vec{\mathbf{L}}\right| - L_0$$

Vector spring force

$$\vec{\mathbf{F}}_{spring} = -k_s s\hat{\mathbf{L}}$$

Gravitational force

See more general form of the gravitational force later.

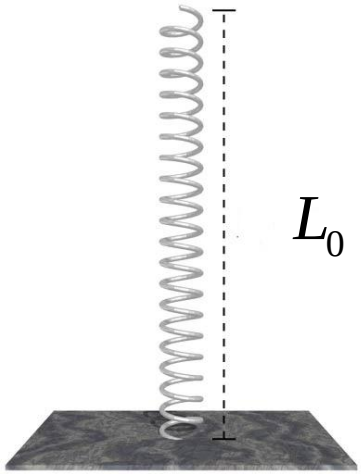
For an object of mass m near the surface of the Earth ...

$$\left|\vec{\mathbf{F}}_{grav}\right| \approx mg$$

where $g = 9.8 \text{ m s}^{-2}$ near the surface of the Earth.

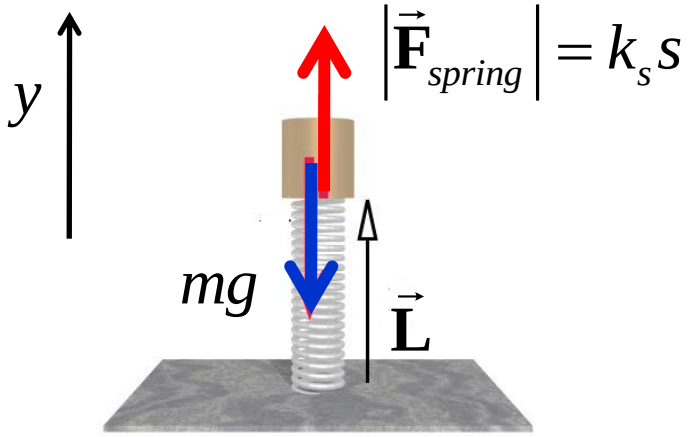
mg is sometimes called the "weight" of the body.

Block on spring 1D : nonconstant net force



$L_0 = 0.2$ m

Spring relaxed and at rest



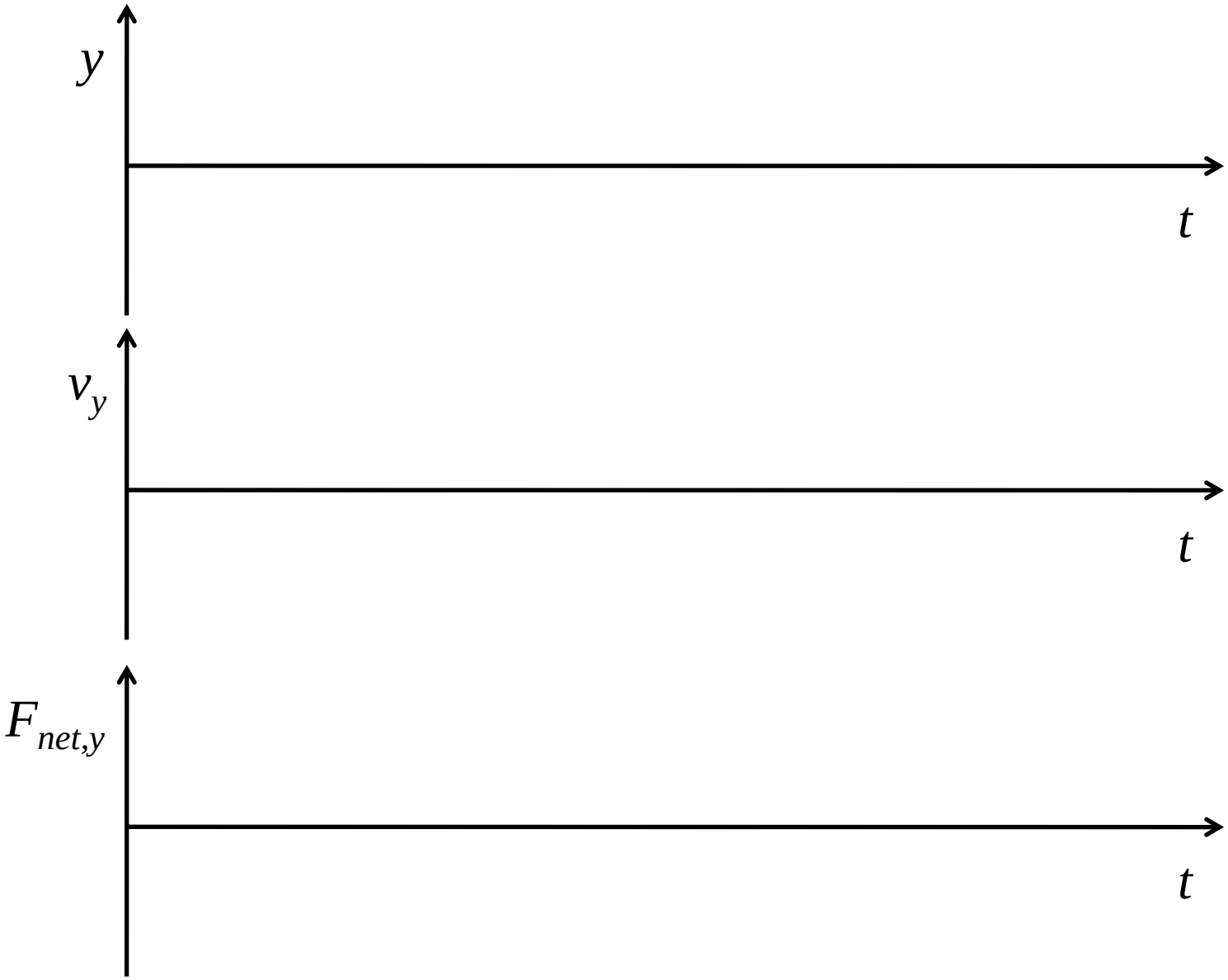
Gently add a block of mass m
Spring is compressed by s

$$k_s s = mg$$

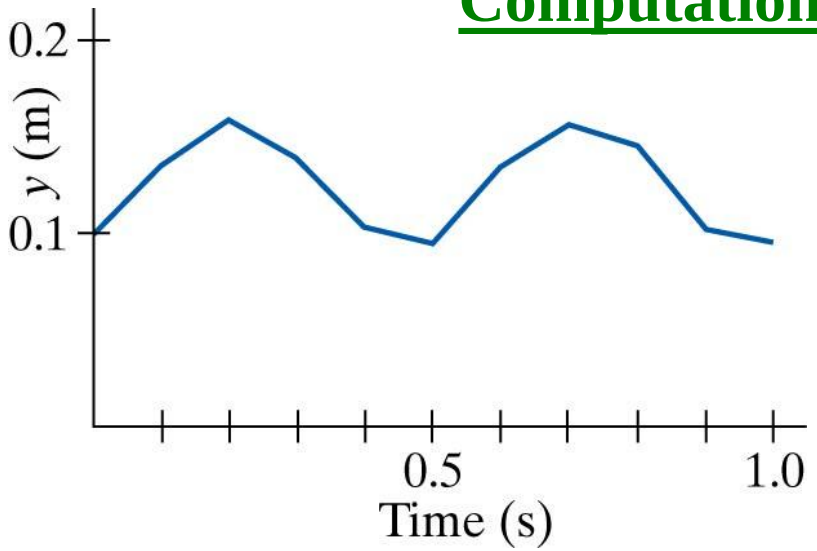
at equilibrium

If you press the block down further (with your hand) and release the spring, then what will happen?

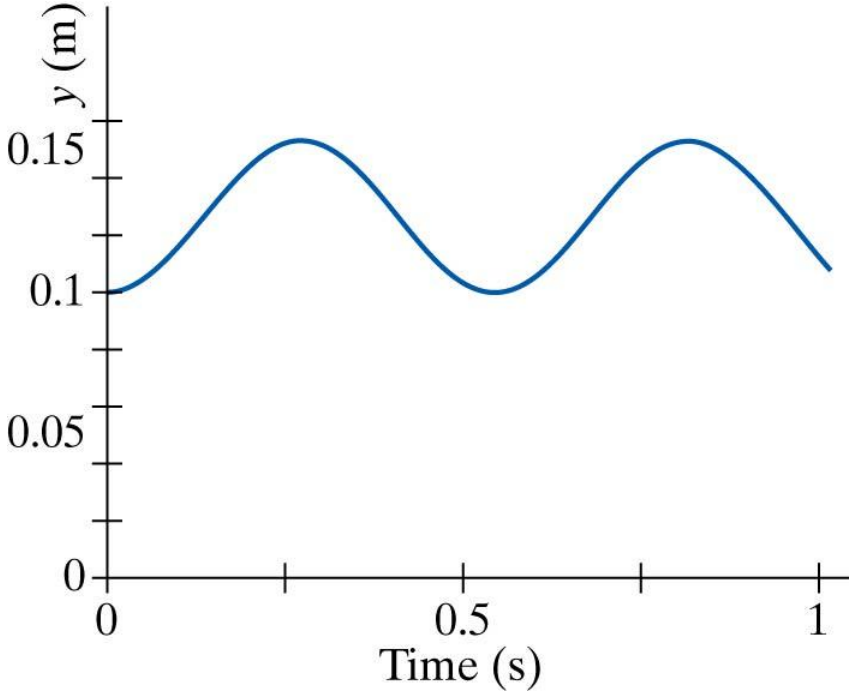
Block on spring 1D : nonconstant net force



Block on spring 1D : nonconstant net force
Computational solution



$\Delta t = 0.1$ s



$\Delta t = 0.01$ s

Special case: constant force

Suppose we have a constant force $\vec{\mathbf{F}}_{av,net} = \vec{\mathbf{F}}_{net}$

In this case the average velocity is also equal to the arithmetic mean of initial and final velocities

$$\vec{\mathbf{v}}_{av} = \frac{\vec{\mathbf{v}}_i + \vec{\mathbf{v}}_f}{2}$$

A solvable "special case".

e.g. object falling in vacuum near surface of planet.
e.g. charged particle in uniform electric field.

Special case: constant force

Solution using momentum principle: $\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i + \vec{\mathbf{F}}_{net}\Delta t$

$$\therefore \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}_{net}}{m}\Delta t$$

$$\therefore \vec{\mathbf{v}}_{av} = \frac{\left(\vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}_{net}}{m}\Delta t\right) + \vec{\mathbf{v}}_i}{2}$$

$$\therefore \vec{\mathbf{v}}_{av} = \vec{\mathbf{v}}_i + \frac{1}{2}\frac{\vec{\mathbf{F}}_{net}}{m}\Delta t$$

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_{av}\Delta t$$

$$\therefore \vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i\Delta t + \frac{1}{2}\frac{\vec{\mathbf{F}}_{net}}{m}\Delta t^2$$

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i \Delta t + \frac{1}{2}\frac{\vec{\mathbf{F}}_{net}}{m}\Delta t^2 \qquad \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}_{net}}{m}\Delta t$$

…or in scalar form:

$$x_f = x_i + v_{i,x}\Delta t + \frac{F_{net,x}}{2m}\Delta t^2 \qquad v_{f,x} = v_{i.x} + \frac{F_{net,x}}{m}\Delta t$$

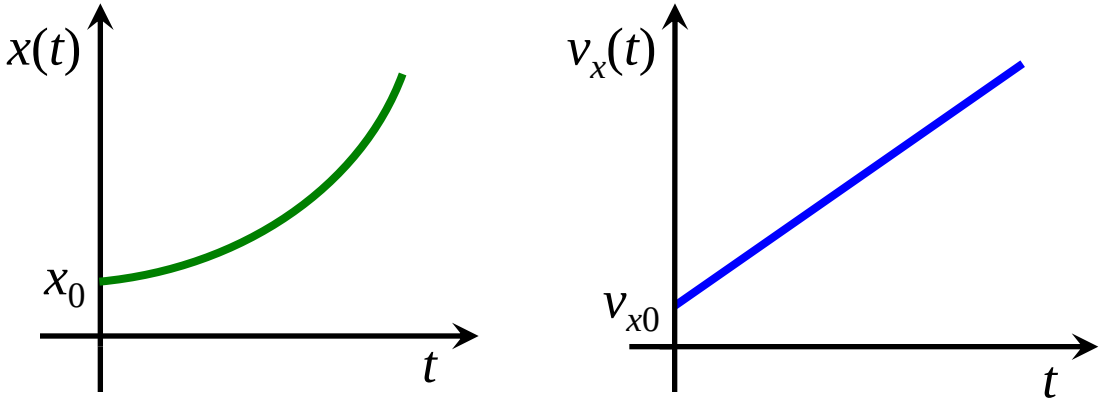
$$y_f = y_i + v_{i,y}\Delta t + \frac{F_{net,y}}{2m}\Delta t^2 \qquad v_{f,y} = v_{i.y} + \frac{F_{net,y}}{m}\Delta t$$

Nature is continuous

The equations of motion arising from Newtonian mechanics

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i \Delta t + \frac{1}{2}\frac{\vec{\mathbf{F}}_{net}}{m}\Delta t^2 \qquad \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}_{net}}{m}\Delta t$$

… allow one to calculate the position and velocity of a particle at any instant in time … they are complete descriptions of the **continuous motion** of the particle …



If one knows the initial conditions, then one is able to predict the motion at some time in the future … the equations are **deterministic** … later on, quantum mechanics showed that nature should be understood as being probabilistic and the determinism of Newton's mechanics applies only to the macro scale.

Other representations of motion

In most cases, when faced with a particular situation to analyze, it is often useful to think about the situation in other ways first, before diving into using (abstract) equations …
… there are a number of **diagrammatic representations** which are often helpful to understand what is going on and to visualize the motion.
It is not sufficient to be able to only calculate the correct answer … you need to also understand the situation fully … which means being able to **describe** the motion in a variety of ways.

The difficulty in using visual-rich representations of a continuous motion is that we need to make choices with respect to what aspects of the motion we want to represent …

Using motion (velocity vector) diagrams

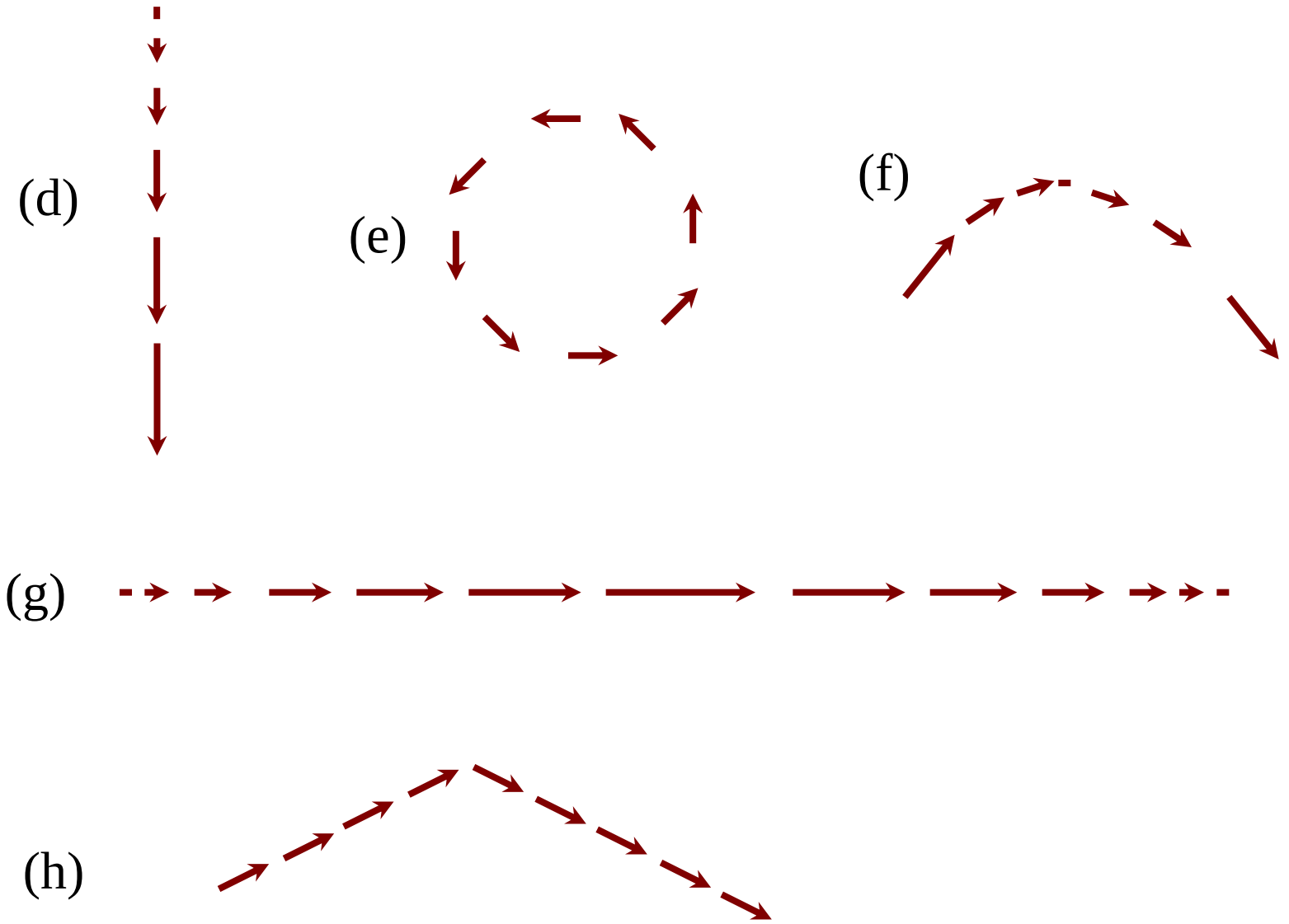
Say that in each case below, a velocity vector was drawn at different positions of a moving object. What can you say about the **net force** acting on the object in each case?

(a) 

(b) 

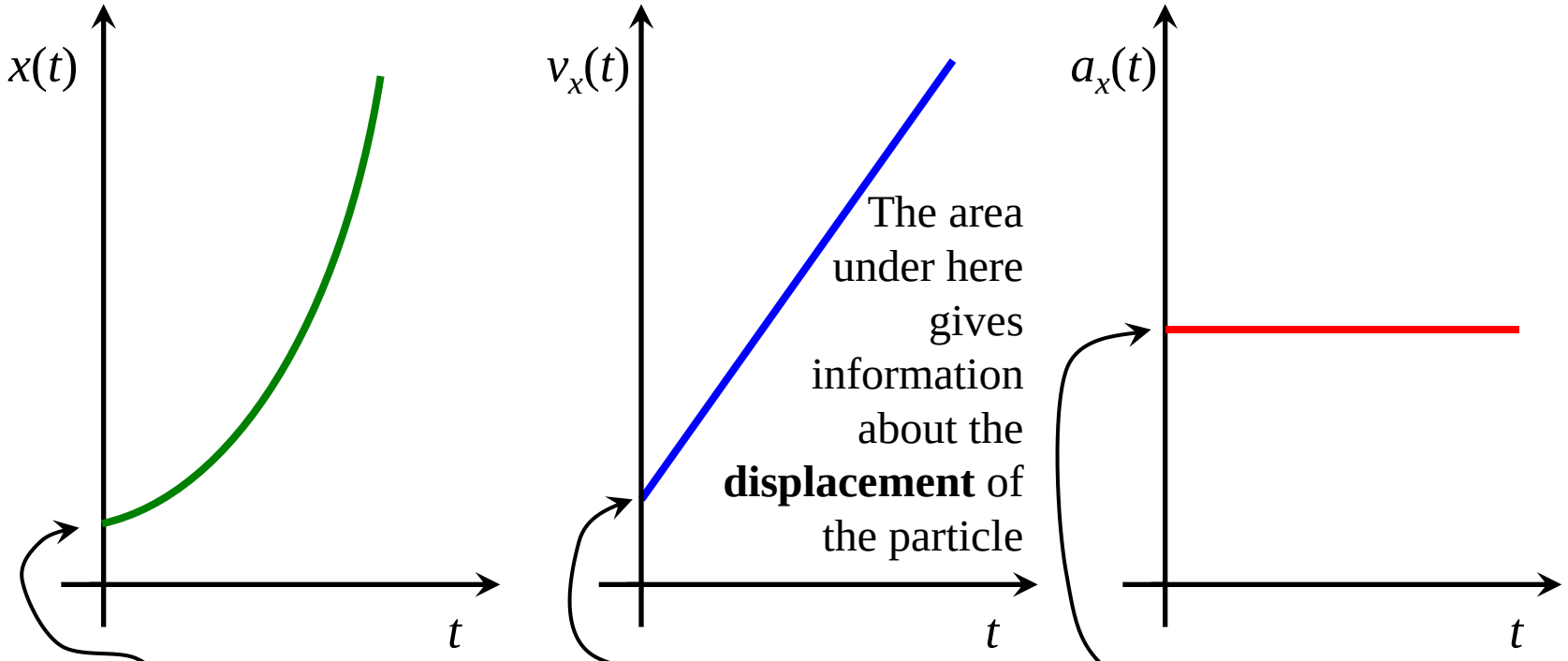
(c) 

More velocity vector representations of motion…



Graphical representations of motion

… for cases of constant (or zero) acceleration



The area under here gives information about the **displacement** of the particle

$$x(t) = x_i + v_{i,x}t + \tfrac{1}{2}a_x t^2$$

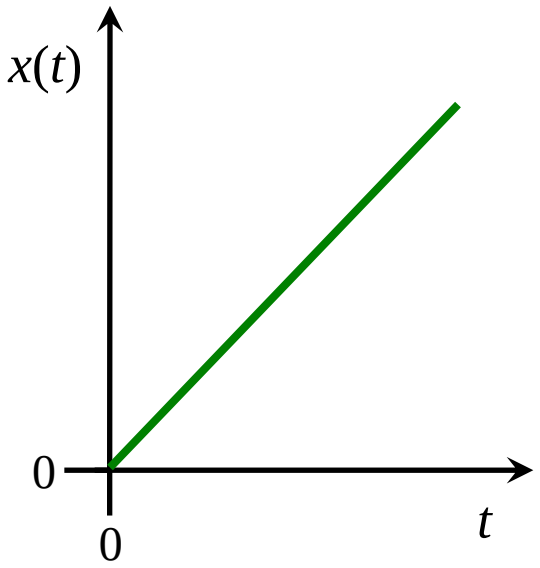
$$\frac{d}{dt}x(t) = \frac{d}{dt}\left(x_i + v_{i,x}t + \tfrac{1}{2}a_x t^2\right) \rightarrow v_x(t) = v_{i,x} + a_x t$$

$$\frac{d}{dt}v_x(t) = \frac{d}{dt}\left(v_{i,x} + a_x t\right) \rightarrow a_x = \text{constant}$$

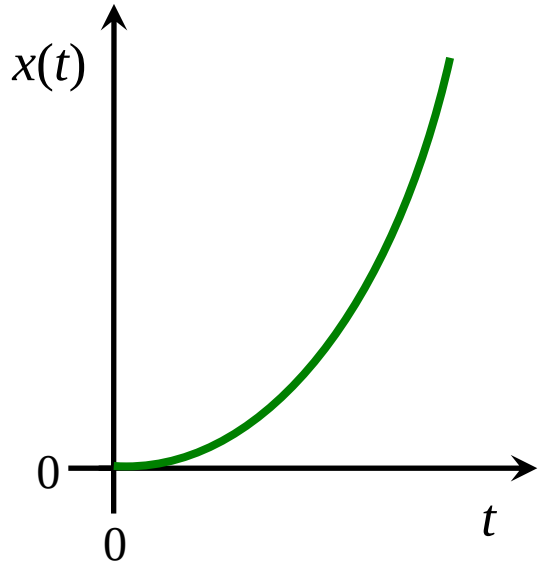
Using graphs of motion … Example 1

Consider the ***x*(*t*) versus time** graphs below, for an object moving in one dimension. In each different case:
(i) provide an example of a situation where the motion of the object is represented by the graph
(ii) sketch the corresponding $v_x(t)$ **vs** ***t*** graph.

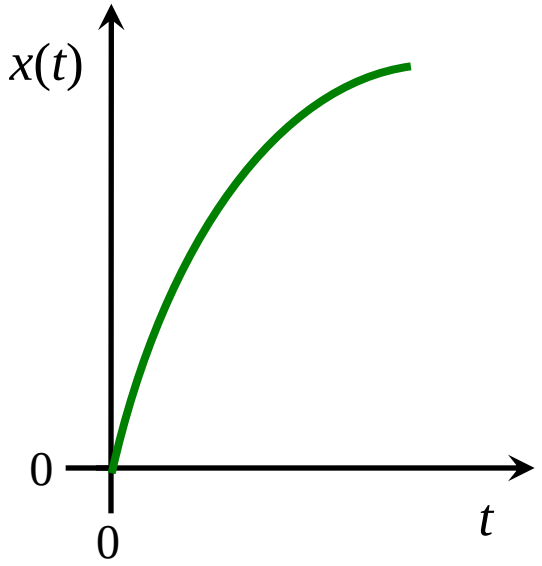
(a)



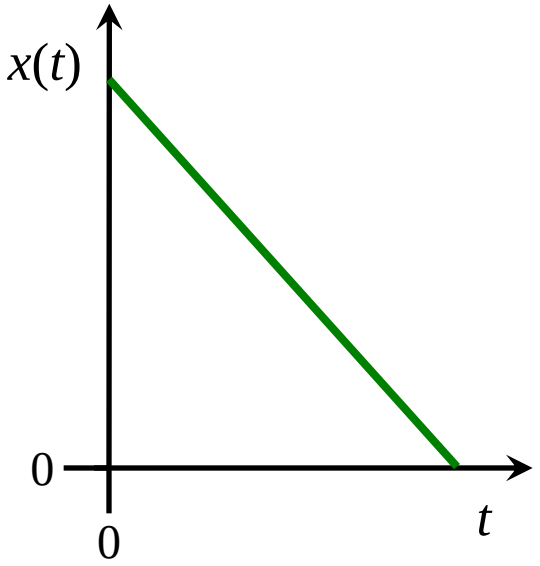
(b)



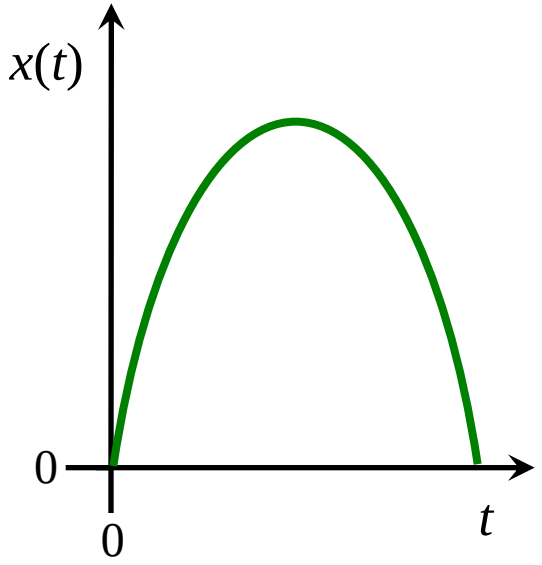
(c)



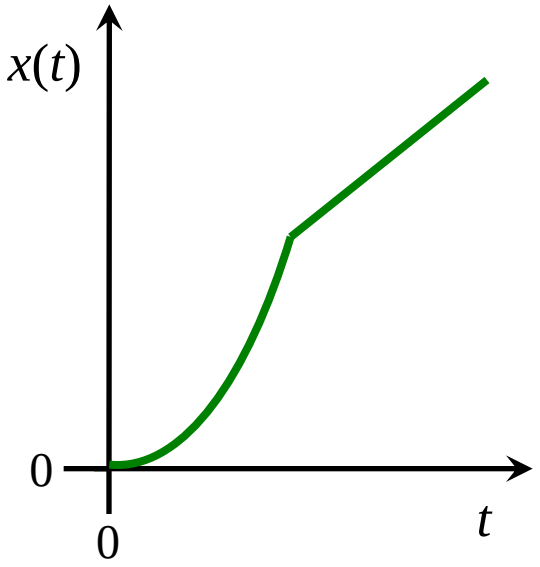
(d)



(e)



(f)

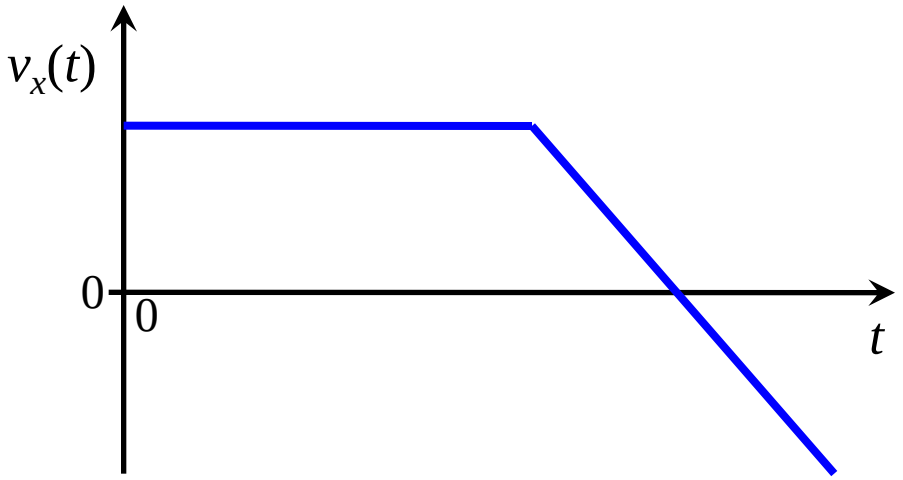


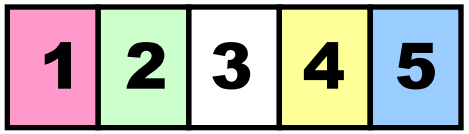
Using graphs of motion … Example 2

Consider the $v_x(t)$ versus time graph below for a car travelling along a flat, straight road.

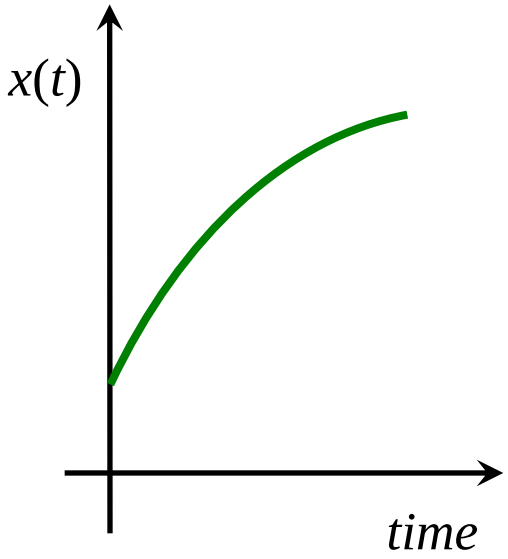
(a) Write down everything you can say about the motion of the car.

(b) Sketch the corresponding ***x*(*t*) vs *t*** and ***a*$_x$(*t*) vs *t*** graphs.





A car moves along a straight road. The graph alongside shows the position of the car as a function of time.

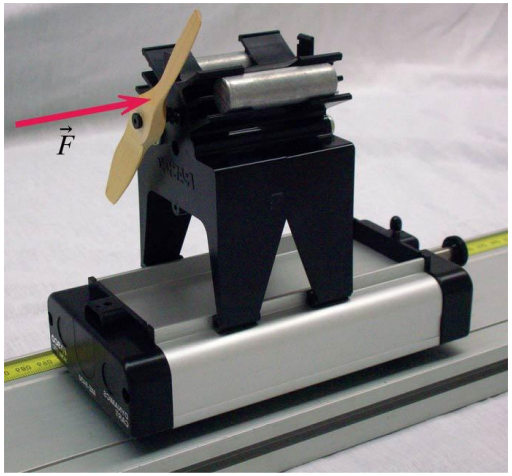


The graph shows that the car:

(A) speeds up all the time
(B) slows down all the time
(C) moves at a constant velocity

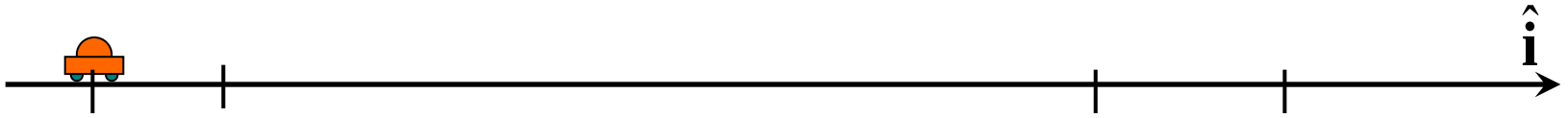
Important worked example of motion in a straight line

A fan cart of mass 0.5 kg is initially at rest. It starts to move under the action of a constant force of 0.375 N and reaches a speed of 15 m s^{-1} after 20 s. It travels at a constant speed for 2 minutes after which time it slows down under the action of a constant force of 0.25 N to stop in 30 seconds.
Assume that the entire motion takes place in a straight line in the $\hat{\mathbf{i}}$ -direction.



Determine the total displacement of the car.

First draw a **picture** of what is happening.
Add in a coordinate system and include all known and unknown variables ...

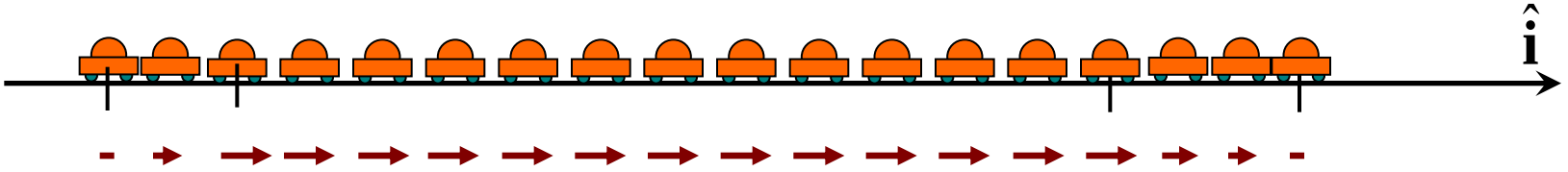


$t = 0$ $t = 20$ s $t = 140$ s $t = 170$ s

$x(t = 0) = 0$ $x(t = 20) = ?$ $x(t = 140) = ?$ $x(t = 170) = ?$

$v_x(t = 0) = 0$ $v(t = 20) = 15$ m s^{-1} $v_x(t = 140) = 15$ m s^{-1} $v_x(t = 170) = 0$ m s^{-1}

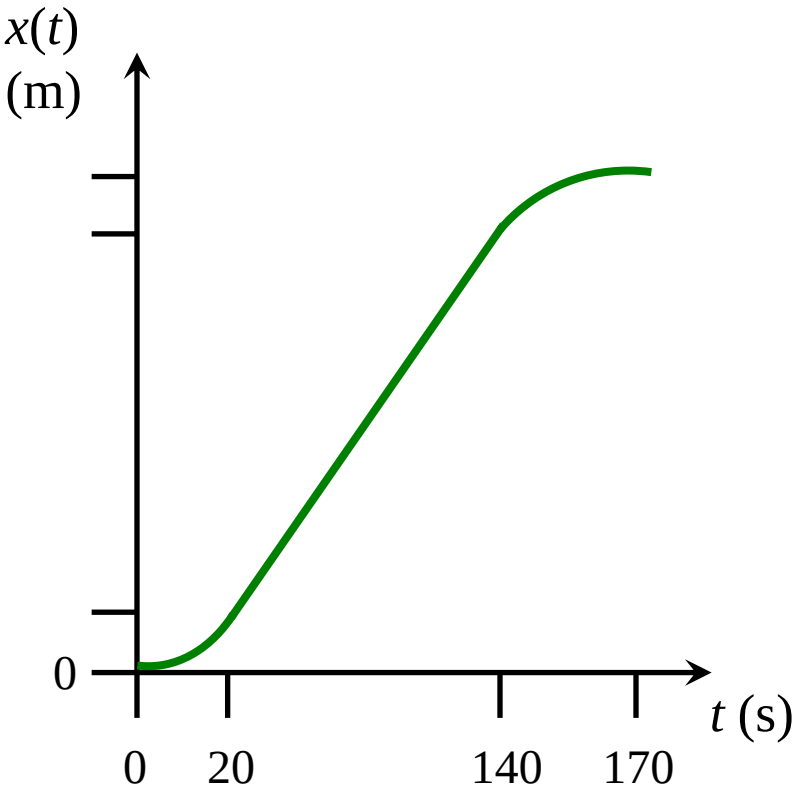
If we look a photograph of the car every 10 s then we would see something like this …



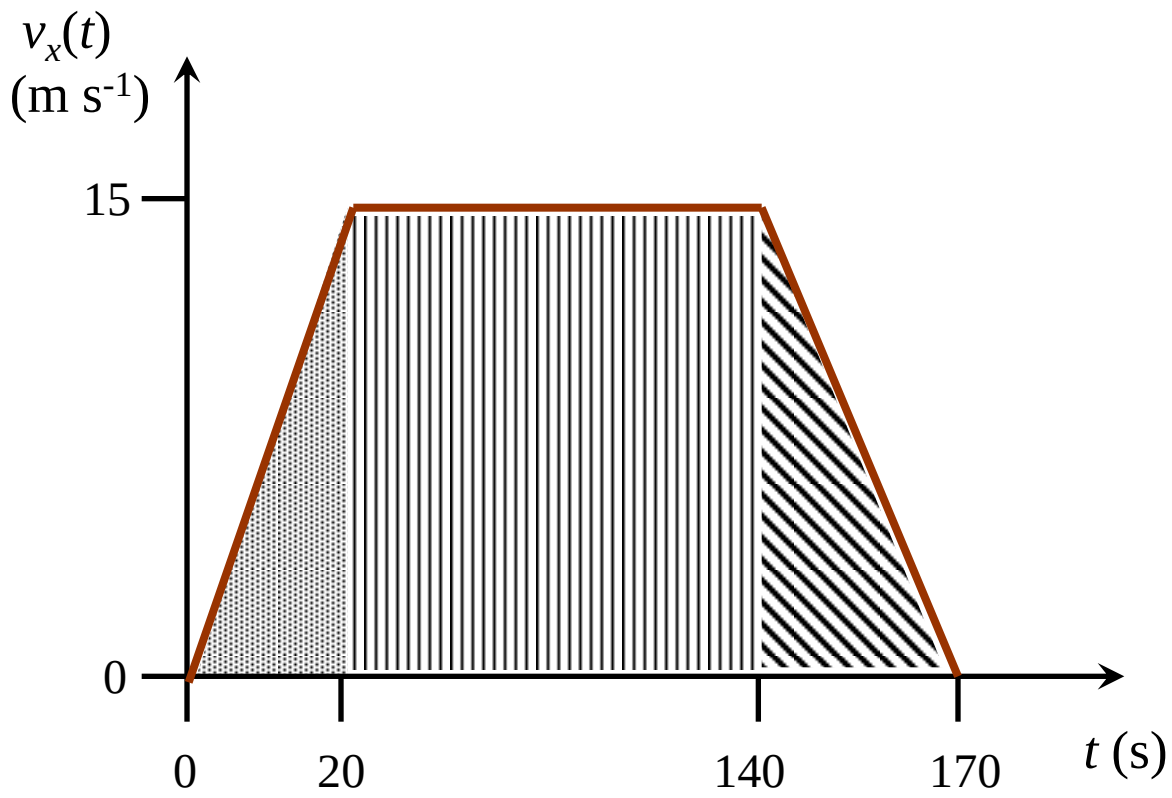
… and drawing the velocity vectors at each position.

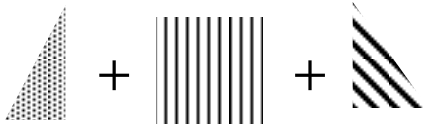
What can you say about the direction of the **net force** acting on the cart in each region?

We can draw graphs of motion …



...and $v_x(t)$ versus t … next slide …



displacement = area under graph =  = $2175\,\hat{\mathbf{i}}$ m

Now the displacements ...

We consider <u>each</u> of the three stages <u>separately</u>.

(i) Between $t = 0$ and $t = 20$ s:

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i \Delta t + \tfrac{1}{2} \frac{\vec{\mathbf{F}}_{net}}{m} \Delta t^2$$

$$\vec{\mathbf{r}}(t = 20) = (0) + (0) + \tfrac{1}{2} \frac{(0.375\hat{\mathbf{i}})}{0.5} (20)^2 = 150\hat{\mathbf{i}} \text{ m}$$

(ii) Between $t = 20$ and $t = 140$ s:

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i \Delta t + \tfrac{1}{2} \frac{\vec{\mathbf{F}}_{net}}{m} \Delta t^2$$

$$\vec{\mathbf{r}}(t = 140) = (150\hat{\mathbf{i}}) + (15\hat{\mathbf{i}})(120) + \tfrac{1}{2}(0)(120)^2 = 1950\hat{\mathbf{i}} \text{ m}$$

(iii) Between $t = 140$ and $t = 170$ s:

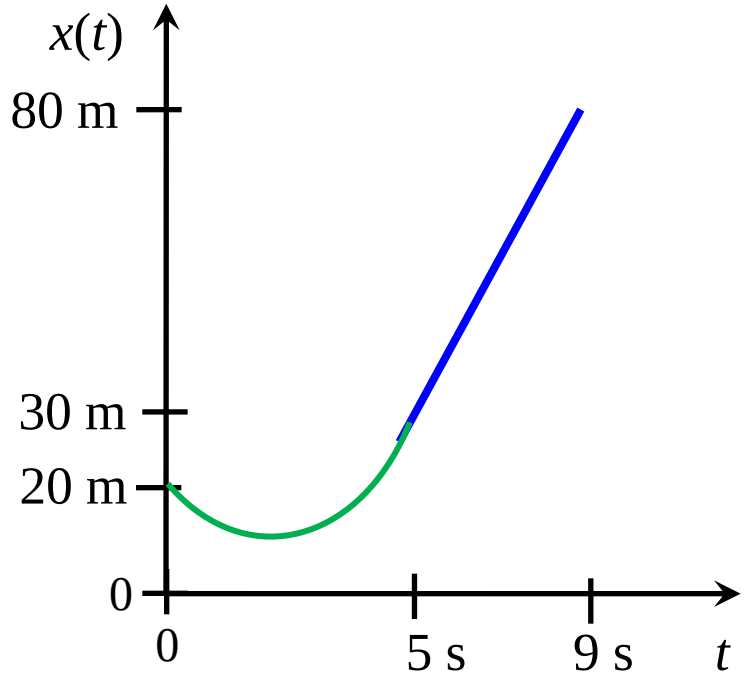
$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i \Delta t + \tfrac{1}{2} \frac{\vec{\mathbf{F}}_{net}}{m} \Delta t^2$$

$$\vec{\mathbf{r}}(t = 170) = (1950\hat{\mathbf{i}}) + (15\hat{\mathbf{i}})(30) + \tfrac{1}{2} \frac{(-0.25\hat{\mathbf{i}})}{(0.5)} (30)^2 = 2175\hat{\mathbf{i}} \text{ m}$$

Final position of the car

Using graphs of motion … Example

The $x(t)$-t graph for the motion of a 3 kg object is shown:

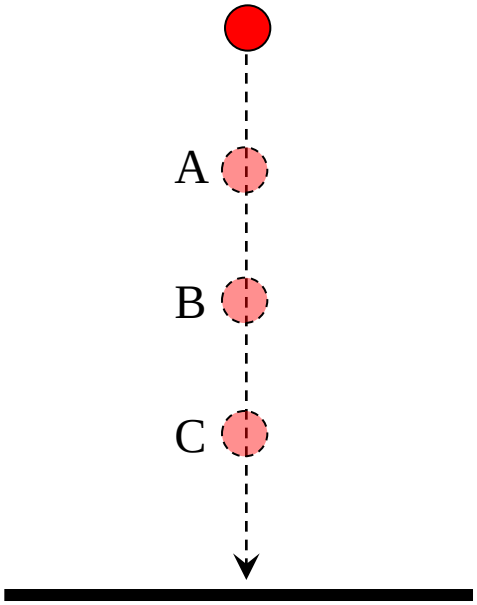


(a) Write down (using words) everything you can say about the motion of the object.

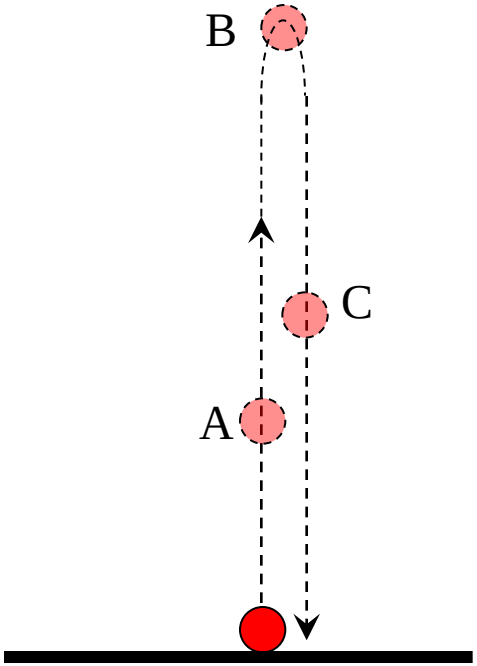
(b) Determine the magnitude and direction of the net force acting on the object during the first 5 seconds.

Bodies in free fall

Consider the following two situations. At each position shown, indicate the magnitude and direction of the **resultant acceleration** of the ball.



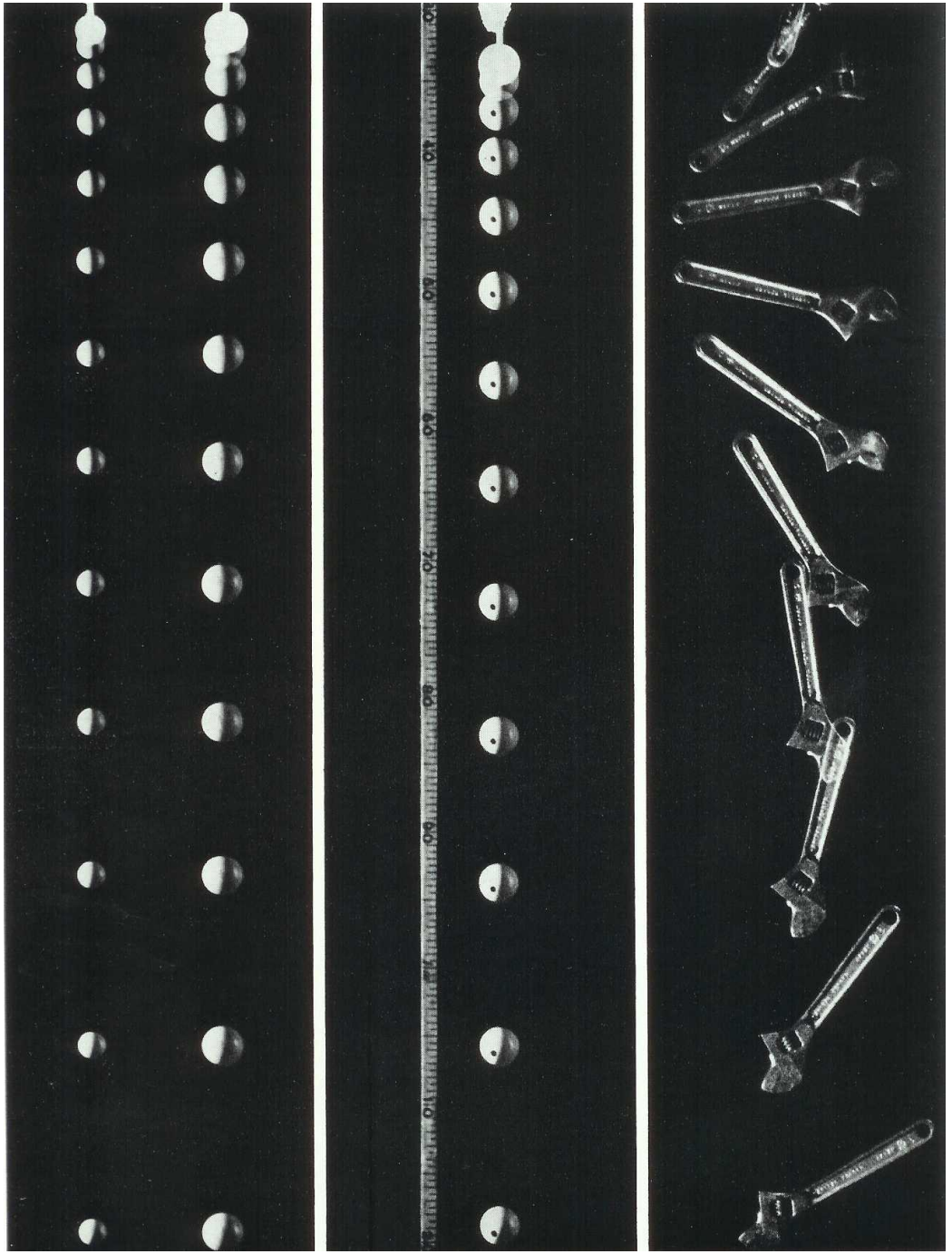
The ball is dropped from rest from a height and allowed to fall to the floor



The ball is thrown upwards, reaches some height, and falls back to the floor

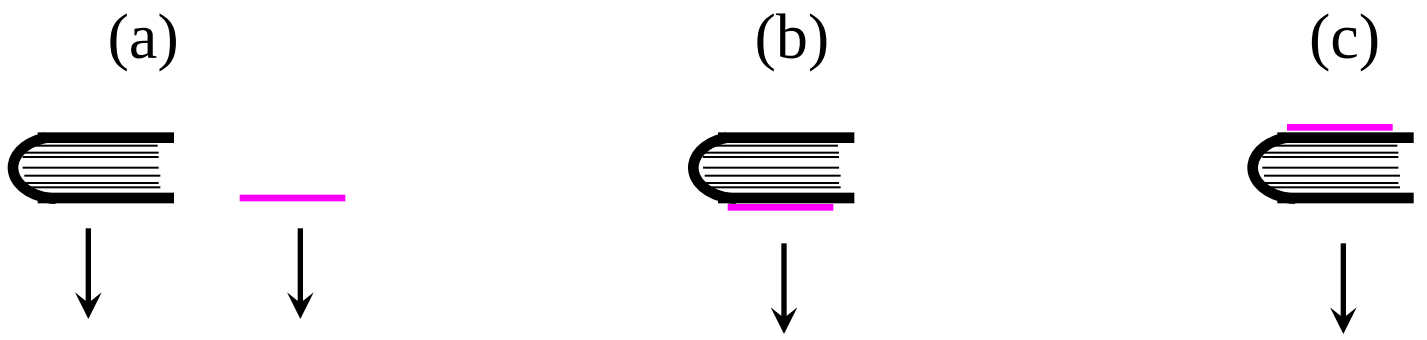
Demonstration

A large ball and a small ball are dropped from the same height in air. Which ball reaches the ground first?



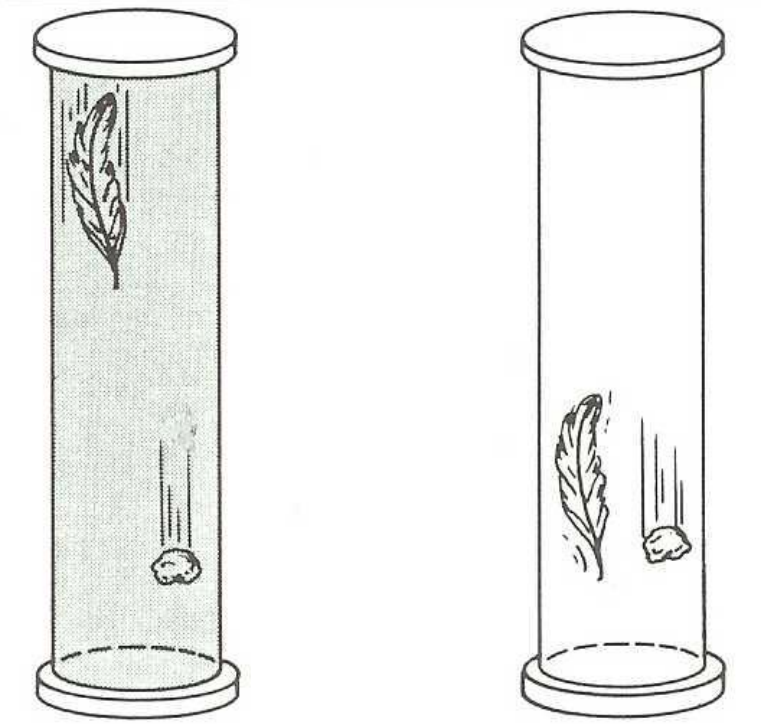
Demonstration

A book and a sheet of paper are dropped from the same height in air. (a) Which reaches the ground first? Does it make a difference if the paper is placed (b) under the book? ... or (c) above the book?



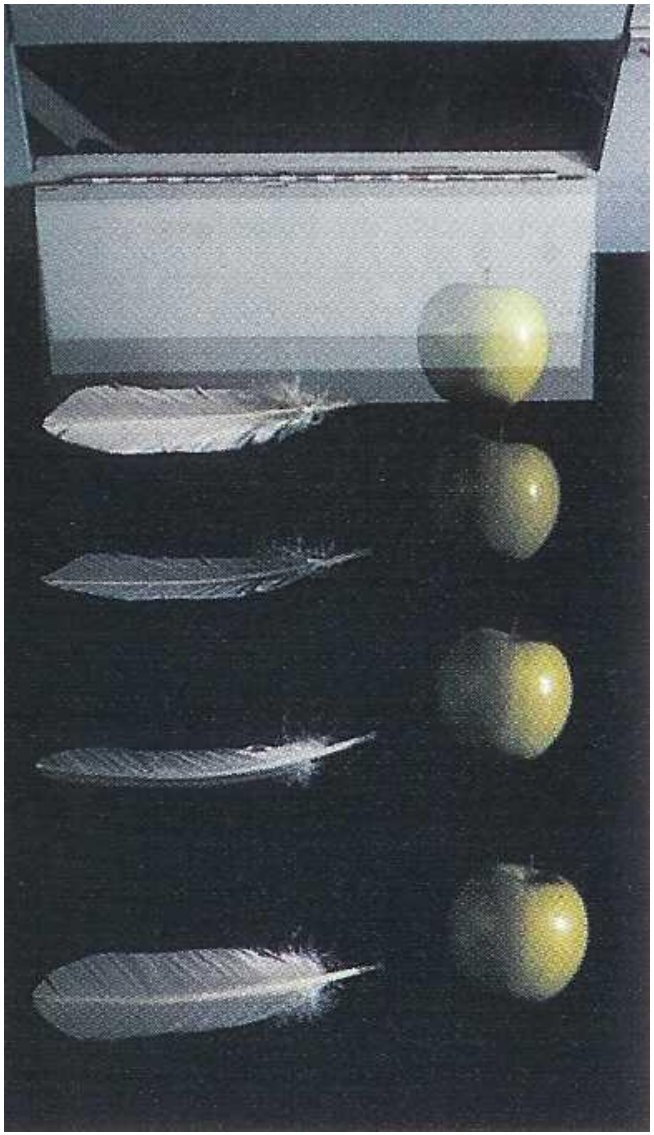
Demonstration

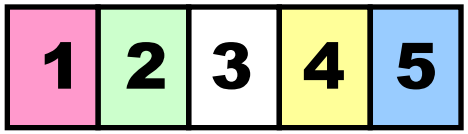
A coin (or a stone) and a disc of paper (or a feather) are dropped from a height in a cylinder filled with air. What will happen if the air is removed from the cylinder?



cylinder is filled with air

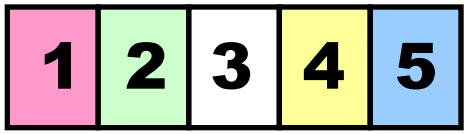
air is removed from the cylinder





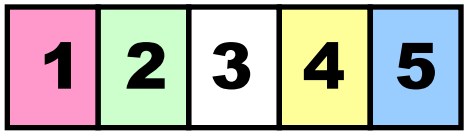
If you **drop** an object in the absence of air resistance, it accelerates downward at 9.8 m s^{-2}. If instead you **throw it downward**, then its downward acceleration after release is

(A) less than 9.8 m s^{-2}
(B) 9.8 m s^{-2}
(C) more than 9.8 m s^{-2}



A ball is initially on the ground, and you kick it with initial velocity < 3, 7, 0 > m/s. At this speed air resistance is negligible. Assume the usual coordinate system.
Which components of the ball's momentum will change in the next half second?

(1) p_x
(2) p_y
(3) p_z
(4) p_x & p_y
(5) p_y & p_z
(6) p_z & p_x
(7) p_x, p_y, & p_z



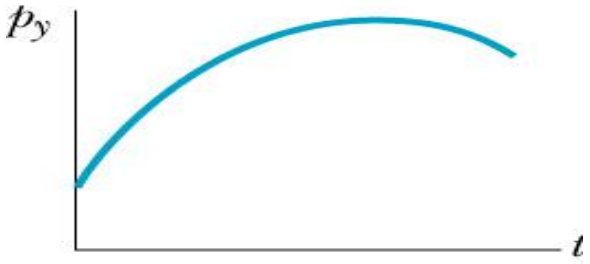
The mass of the ball is 500 g, and its initial velocity is < 3, 7, 0 > m/s. What is the net impulse acting on the ball during the next 0.5 seconds after you kicked it?

(1) < 0, 2.45, 0 > N s
(2) < 0, –2.45, 0 > N s
(3) < 0, 9.8, 0 > N s
(4) < 0, –9.8, 0 > N s
(5) < 0, 4.9, 0 > N s
(6) < 0, –4.9, 0 > N s

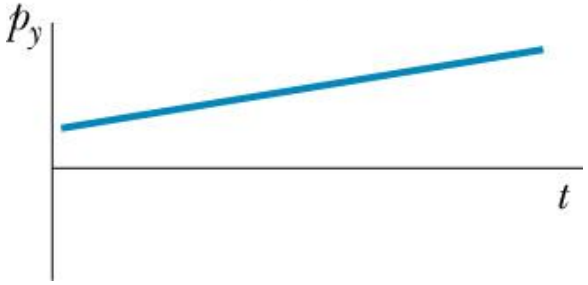
1	2	3	4	5

Which graph correctly shows p_y for the ball during this 0.5 s?

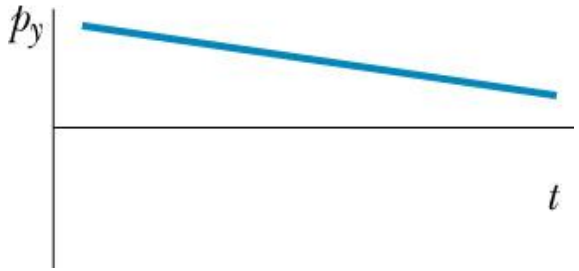
(1)



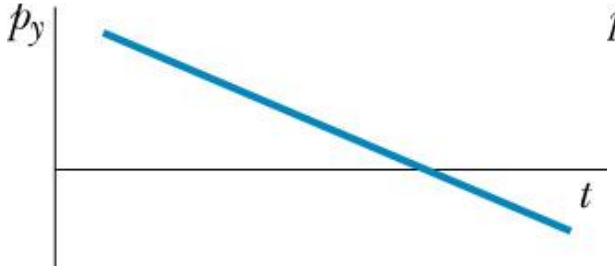
(2)



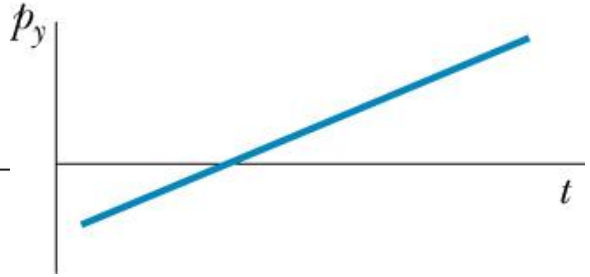
(3)



(4)



(5)



(6)



Example: Throwing a ball upwards

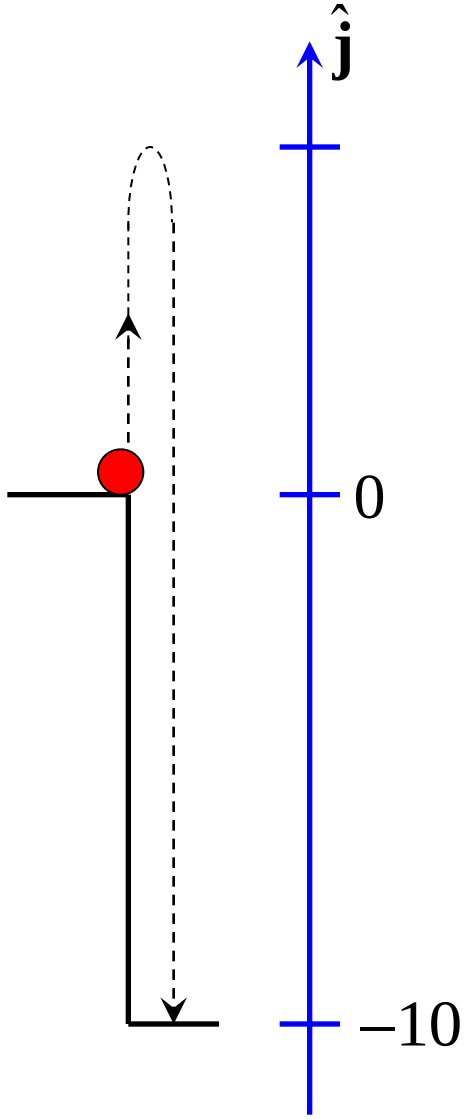
Say that you throw a ball vertically upward at 7 m s^{-1} from a balcony that is 10 m above the ground. After reaching a maximum height, the ball drops past the balcony, to the ground.

We choose a coordinate system with the $\hat{\mathbf{j}}$ direction upwards and the origin at the balcony.

Complete the freeze frame representation of the motion of the ball …

… and also draw a **velocity vector** at each of these positions of the ball.

What is the acceleration of the ball at each of the positions shown?



Say that you throw a ball vertically upward at 7 m s^{-1} from a balcony that is 10 m above the ground. After reaching a maximum height, the ball drops past the balcony, to the ground.
With reference to the coordinate axis given …

Initial position of the ball =

Final position of the ball =

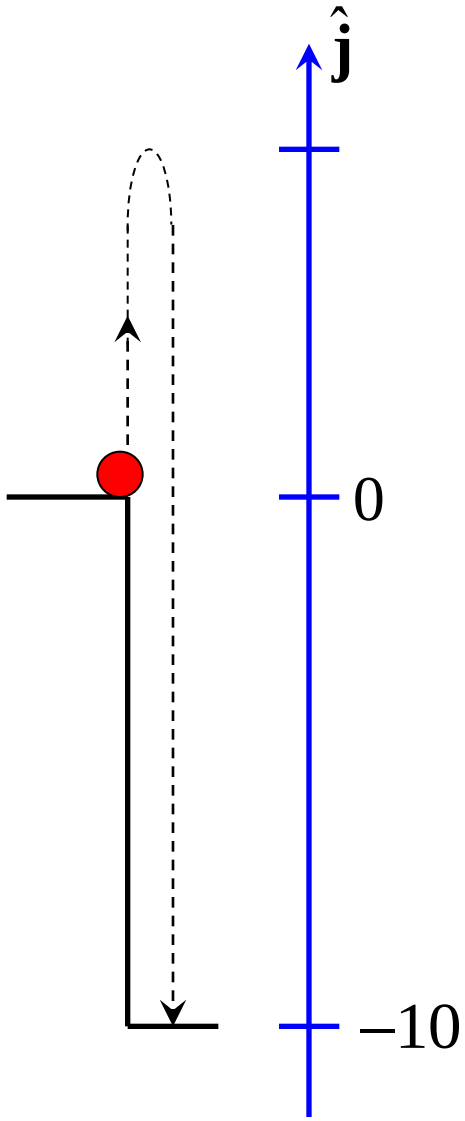
Displacement of the ball when it reaches the ground =

Initial velocity of the ball =

Acceleration of the ball while traveling upward =

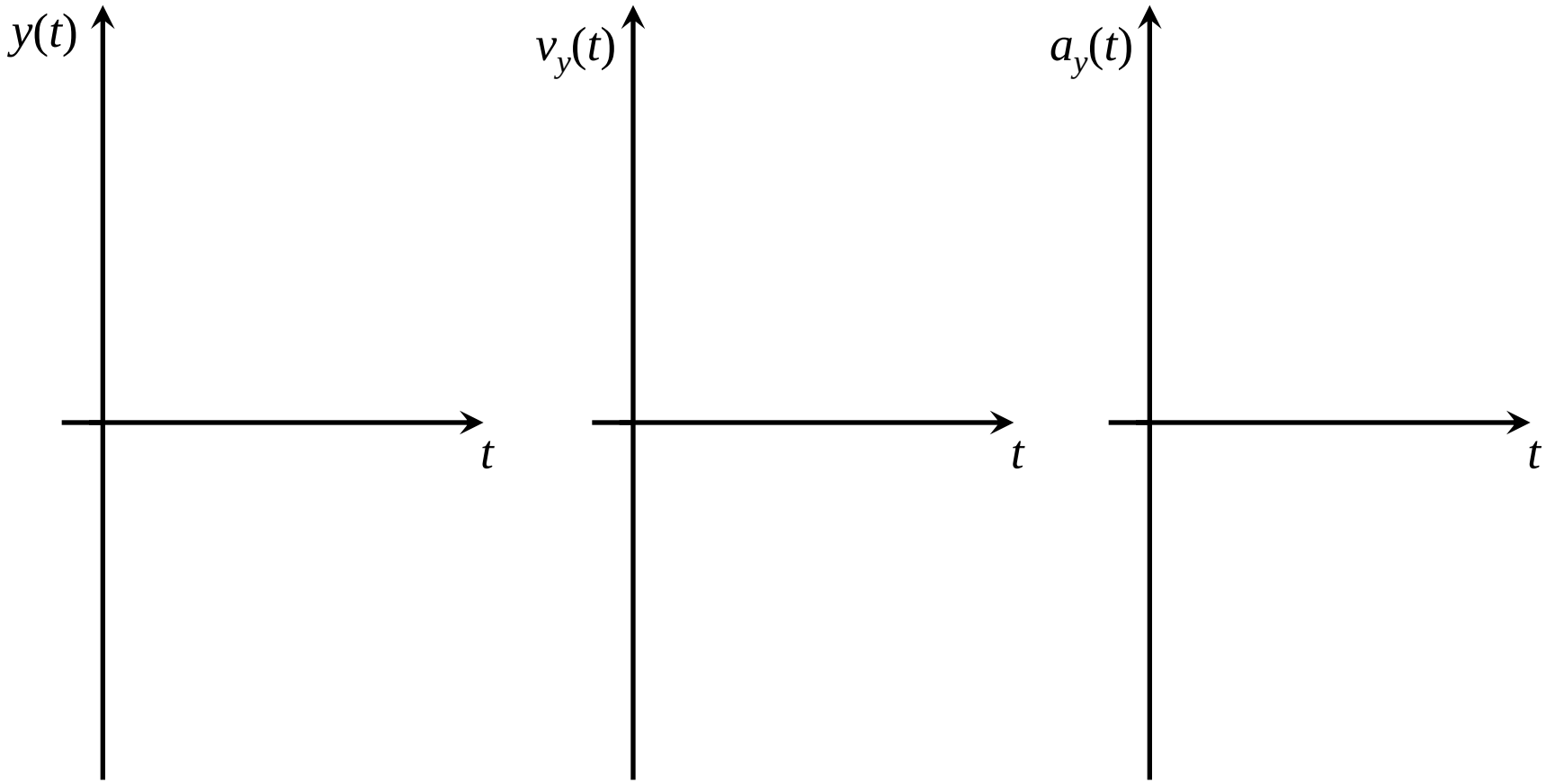
Acceleration of the ball while traveling downward =

Acceleration of the ball at maximum height =

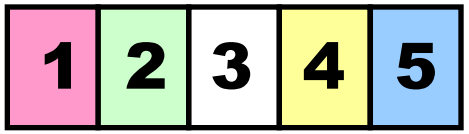


Say that you throw a ball vertically upward at 7 m s^{-1} from a balcony that is 10 m above the ground. After reaching a maximum height, the ball drops past the balcony, to the ground.

Draw the three **graphs of motion** …

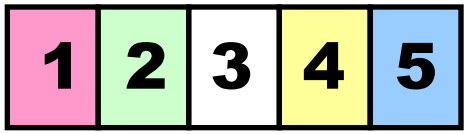


Say that you throw a ball vertically upward at 7 m s^{-1} from a balcony that is 10 m above the ground. After reaching a maximum height, the ball drops past the balcony, to the ground.
Now determine how long it takes for the ball to reach the ground.



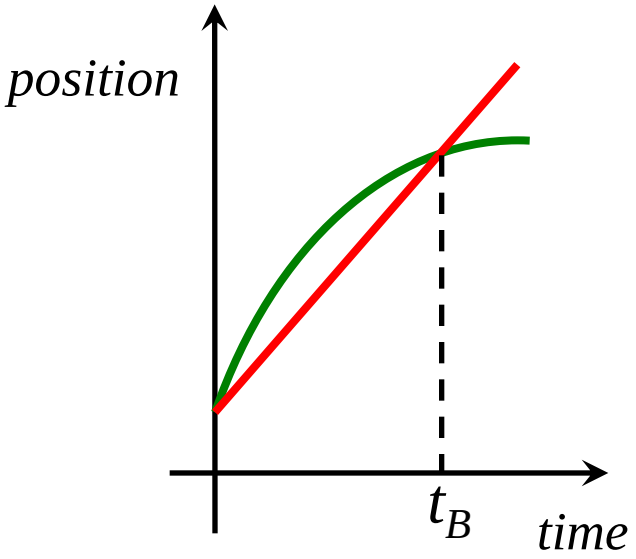
A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed.
Neglecting air resistance, the ball to hit the ground below the cliff with the greater speed is the one initially thrown

(A) upward
(B) downward
(C) neither - they both hit at the same speed.



The graph shows position as a function of time for two trains running on parallel tracks.
Which is true:

(A) At time t_B both trains have the same velocity
(B) Both trains speed up all the time
(C) Both trains have the same velocity at some time before t_B



Example 3

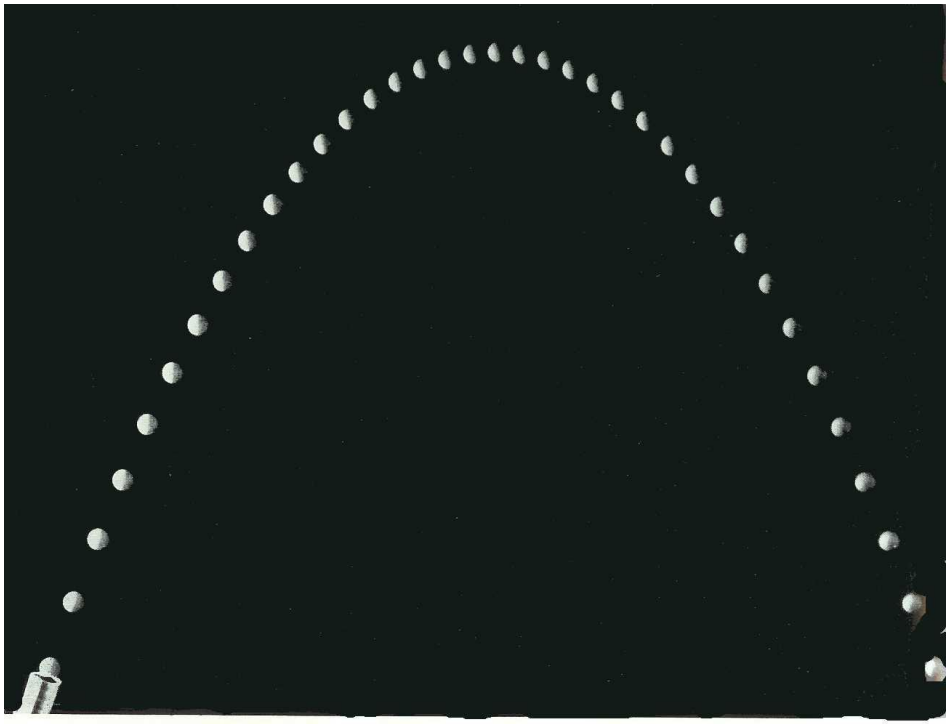
You and your brother are playing a game on the beach. He is standing on a ledge 15 m above you (standing on the beach) and throws a ball vertically upward with an initial speed of 30 m s^{-1}. The idea is for you to simultaneously throw a stone vertically upward from the beach so that it hits the ball at the apex of its flight. At what speed should you throw the stone?

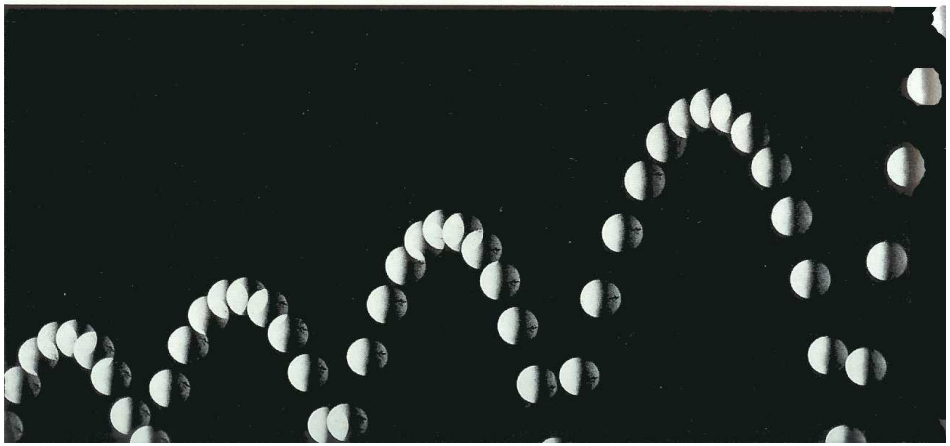
Example 4

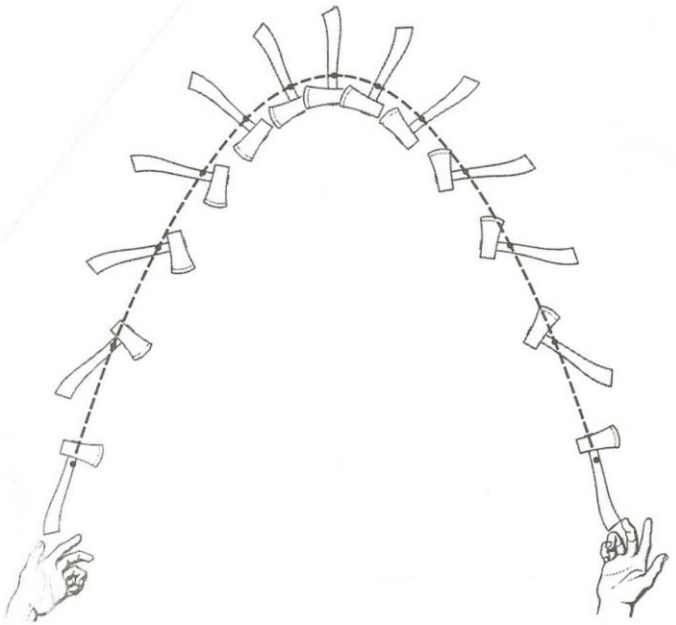
A passenger train and goods locomotive are travelling in the **same** direction along the same track. The passenger train is travelling at 100 km/h and goods locomotive at 18 km/h. The driver of the passenger train is shocked to see the goods locomotive 420 m ahead of him and immediately hits the brakes. The passenger train then immediately starts to slow down with a constant acceleration.

(a) Calculate what the acceleration of the passenger train needs to be in order for it **to just touch** the goods locomotive (and not smash into it).
(b) Sketch, on the same axes, the position-time, velocity-time and acceleration-time graphs for the passenger train and the goods locomotive. Label each carefully.

Projectiles

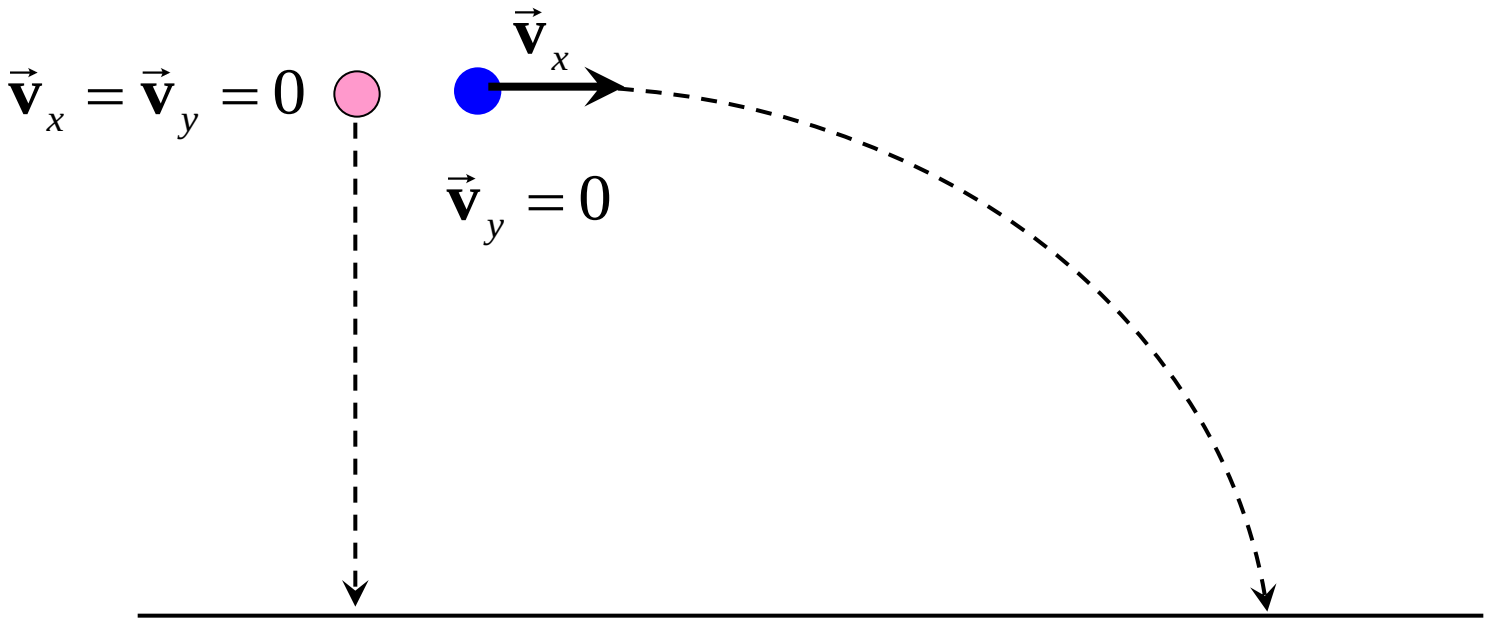


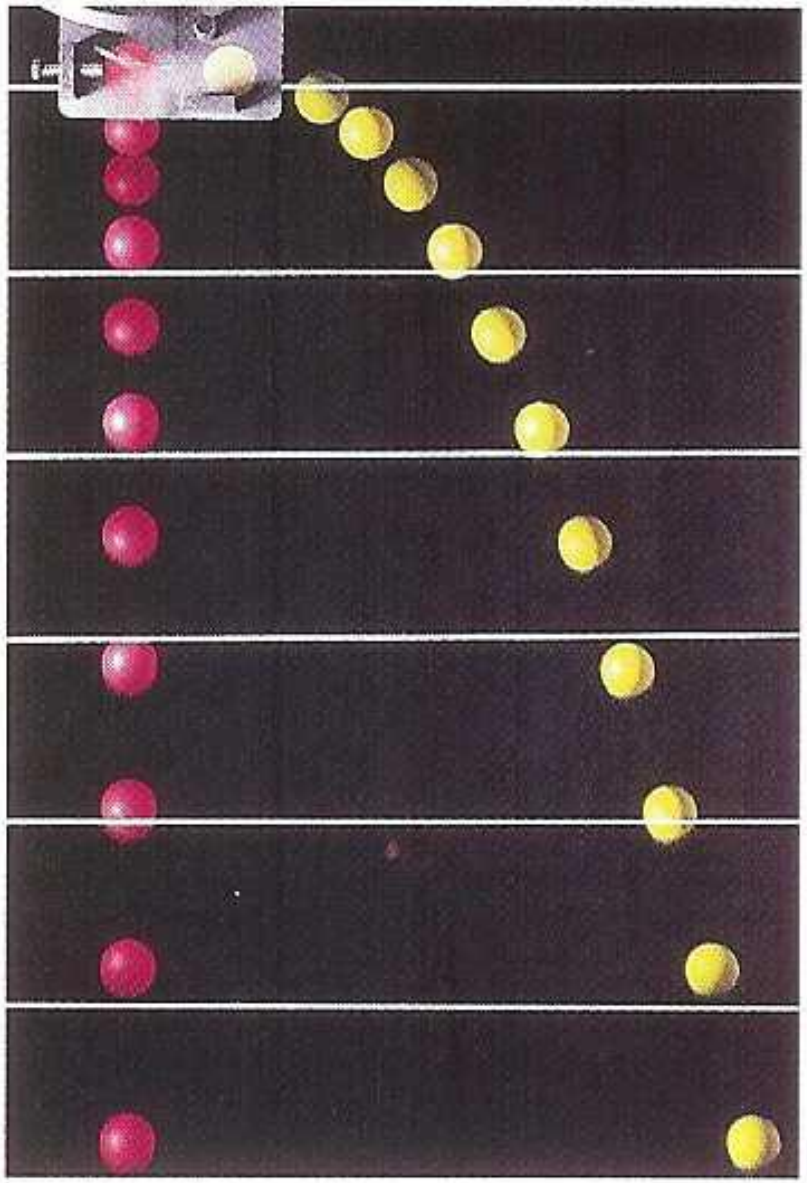




Demonstration

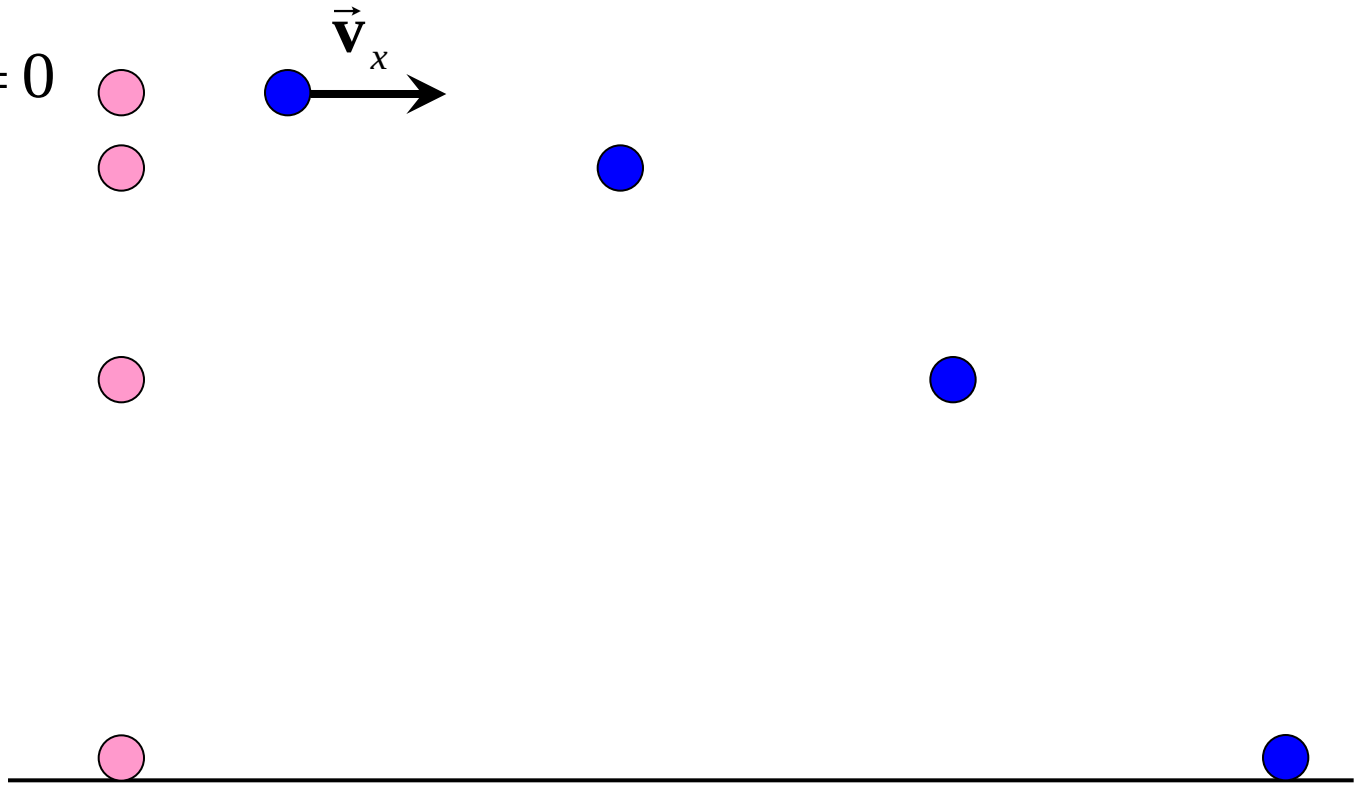
A ball in dropped from a height. At the same instant, a second ball is projected horizontally from the same height.
Which ball hits the ground first?



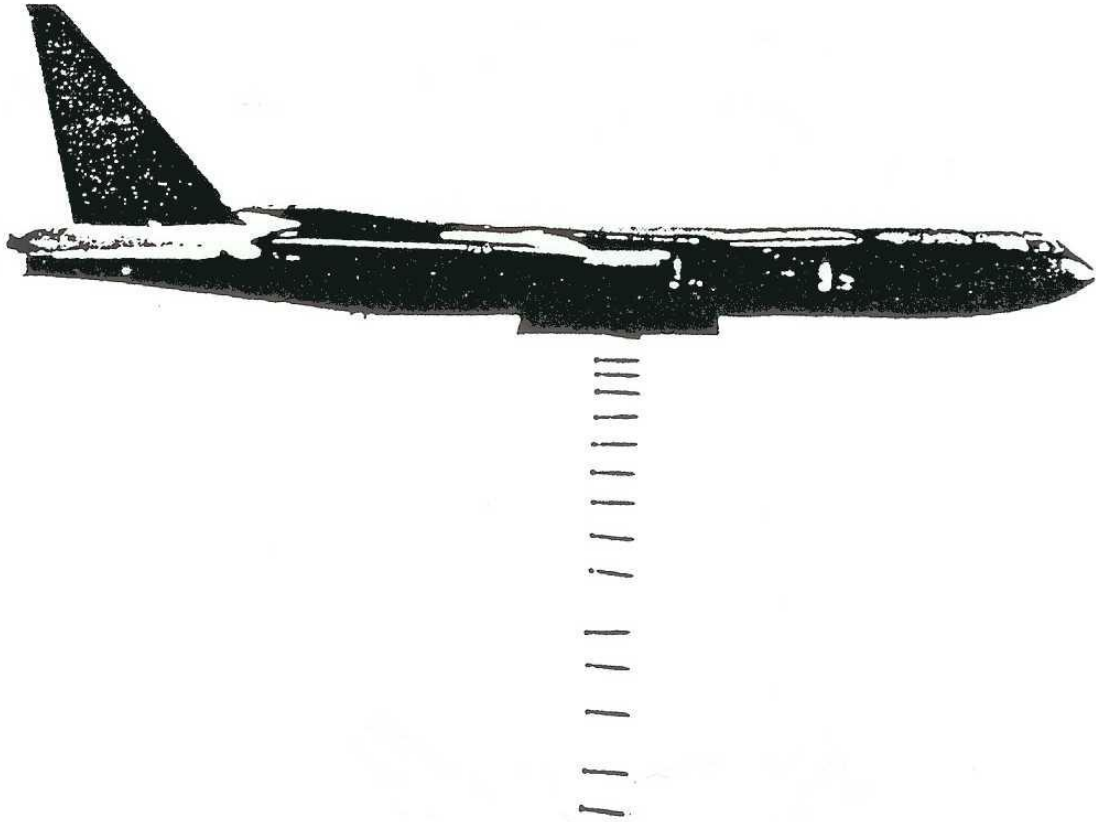


At each position shown, draw in the *x* and *y* components of the velocity vectors of both balls.

$\vec{\mathbf{v}}_x = \vec{\mathbf{v}}_y = 0$



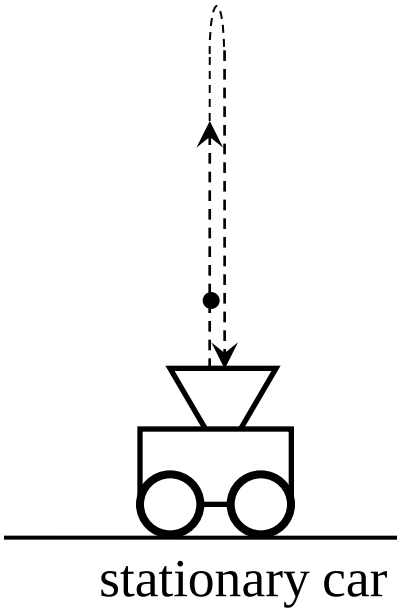
What is the magnitude and direction of the acceleration of each ball at each position shown?



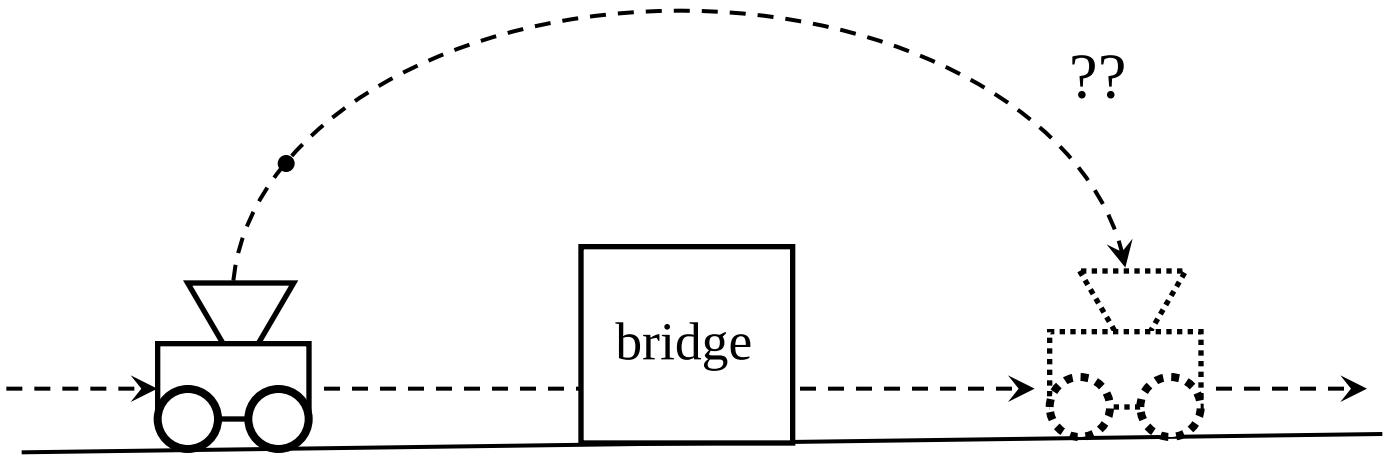
Demonstration

Rolling cart catches own ball

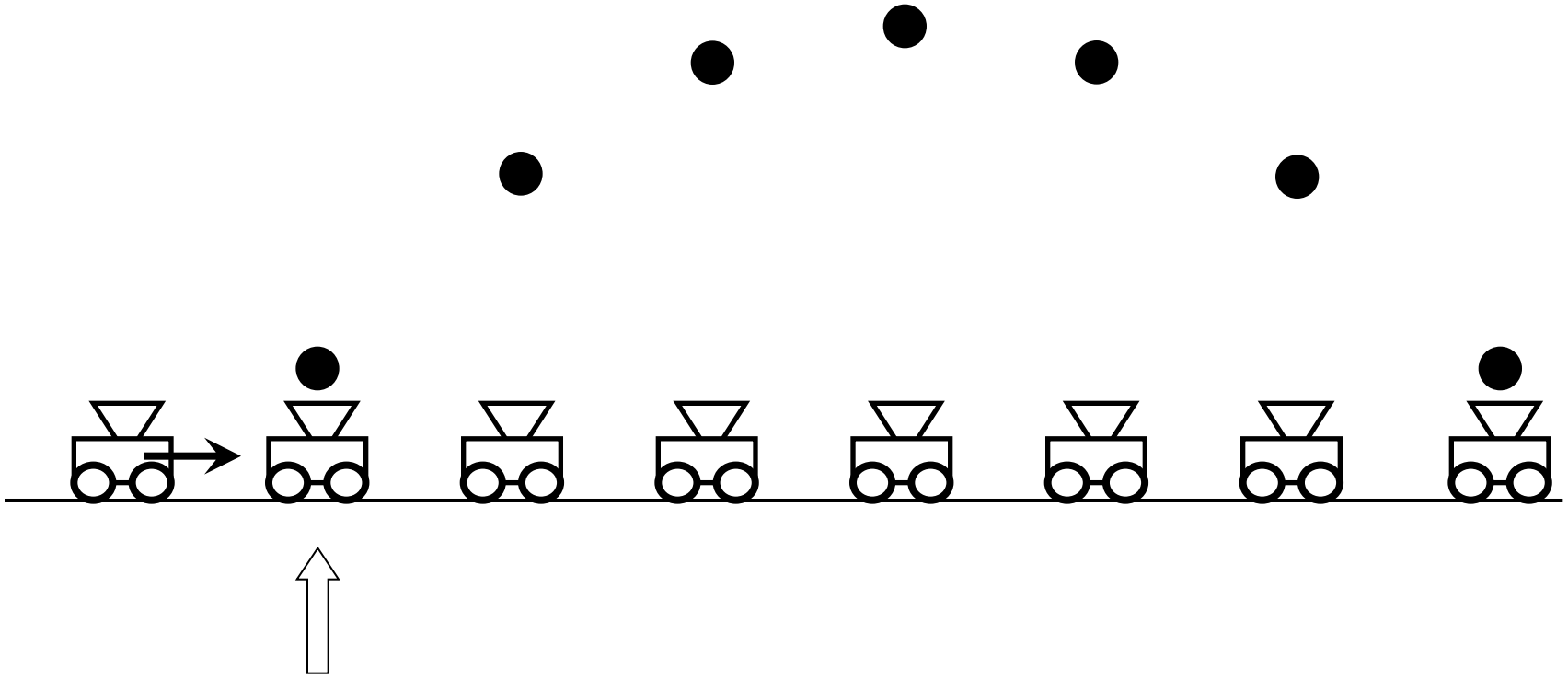
A special car is designed with low friction axles and has a spring-loaded mechanism which projects a small steel ball vertically upward.
A needle attached to a string is used to release the ball and an upward-facing funnel catches the ball.



What will happen if the ball is fired upward while the car is moving horizontally at a constant speed? Where will the ball land?

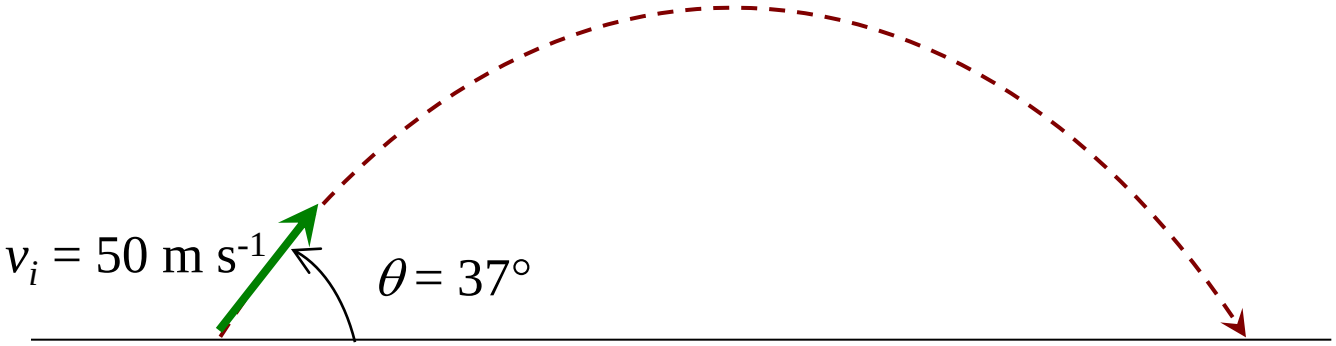


At each position shown, draw in the *x* and *y* components of the velocity vectors of the car and the ball.



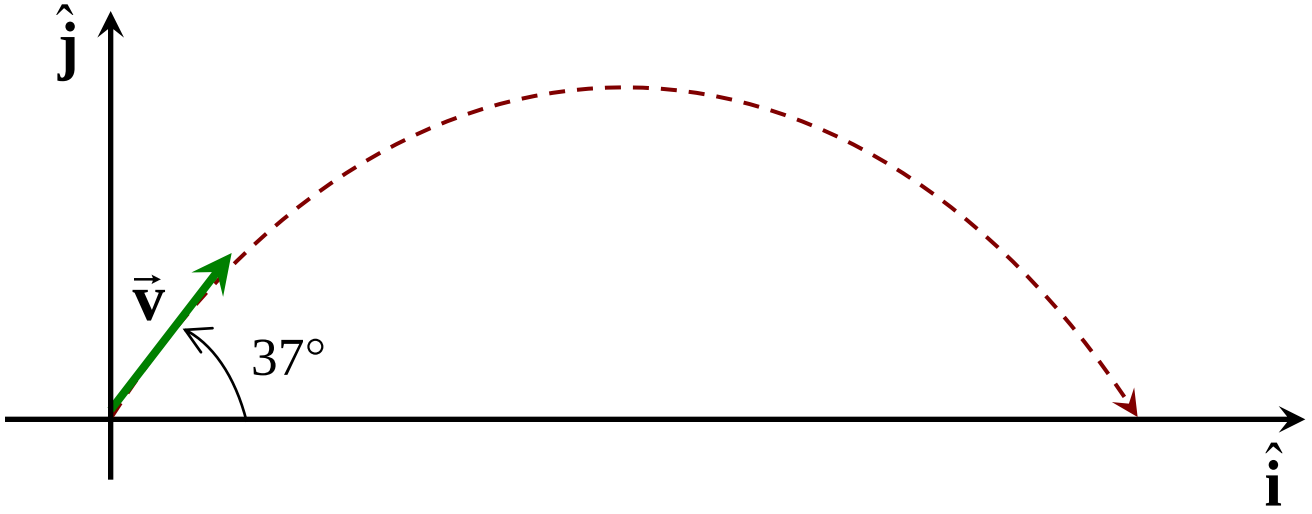
Example

A ball is projected from the origin with initial velocity having magnitude 50 m s^{-1} at an angle of 37° to the horizontal. We are interested in knowing the position and velocity of the ball as a function of time.



$$\vec{\mathbf{v}}_{i,x} = v_i \cos\theta \hat{\mathbf{i}}$$

$$\vec{\mathbf{v}}_{i,y} = v_i \sin\theta \hat{\mathbf{j}}$$



We apply the same equations of motion…

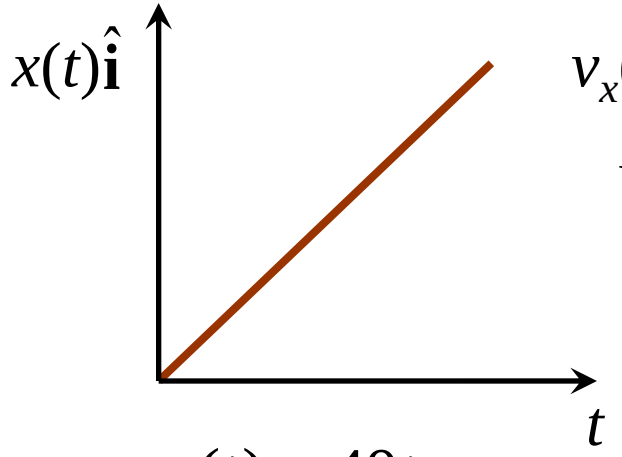
$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i \Delta t + \tfrac{1}{2}\frac{\vec{\mathbf{F}}_{net}}{m}\Delta t^2 \qquad \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}_{net}}{m}\Delta t$$

$$\vec{\mathbf{r}}_f = \langle 0,0,0\rangle + \langle 50\cos 37°, 50\sin 37°, 0\rangle \Delta t + \tfrac{1}{2}\frac{\langle 0,-mg,0\rangle}{m}\Delta t^2$$

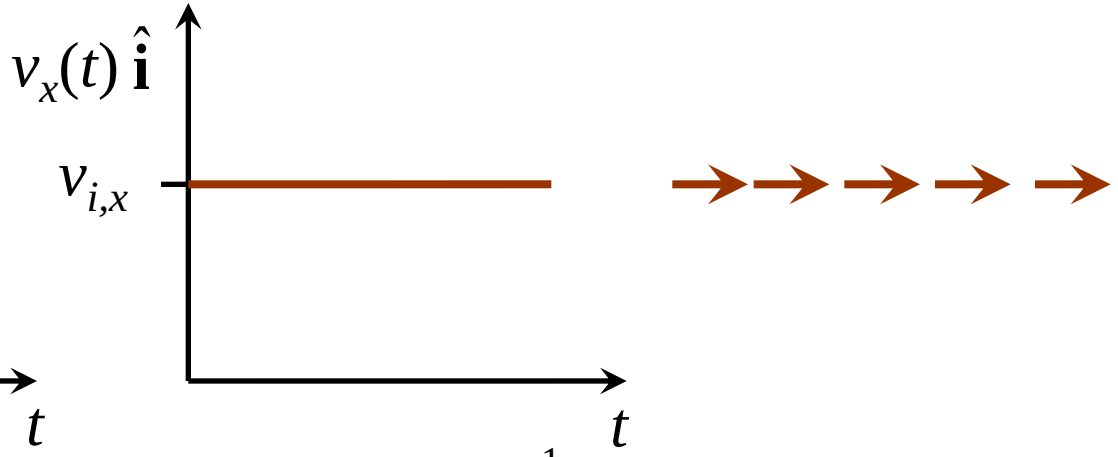
$$\vec{\mathbf{v}}_f = \langle 50\cos 37°, 50\sin 37°, 0\rangle + \frac{\langle 0,-mg,0\rangle}{m}\Delta t$$

Complete the table below by calculating the components of the position and velocity vectors for this projectile.

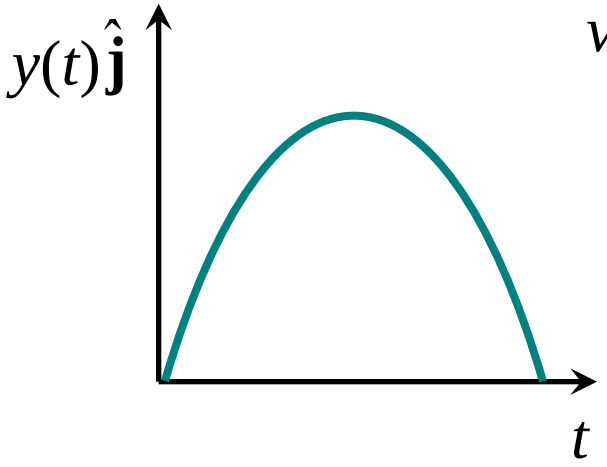
time (s)	x (m)	y (m)	v_x (m s^{-1})	v_y (m s^{-1})
0	0	0	40	30
1	40	25	40	20
2	80	40	40	10
3	120	45	40	0
4	160	40	40	–10
5	200	25	40	– 20
6	240	0	40	– 30



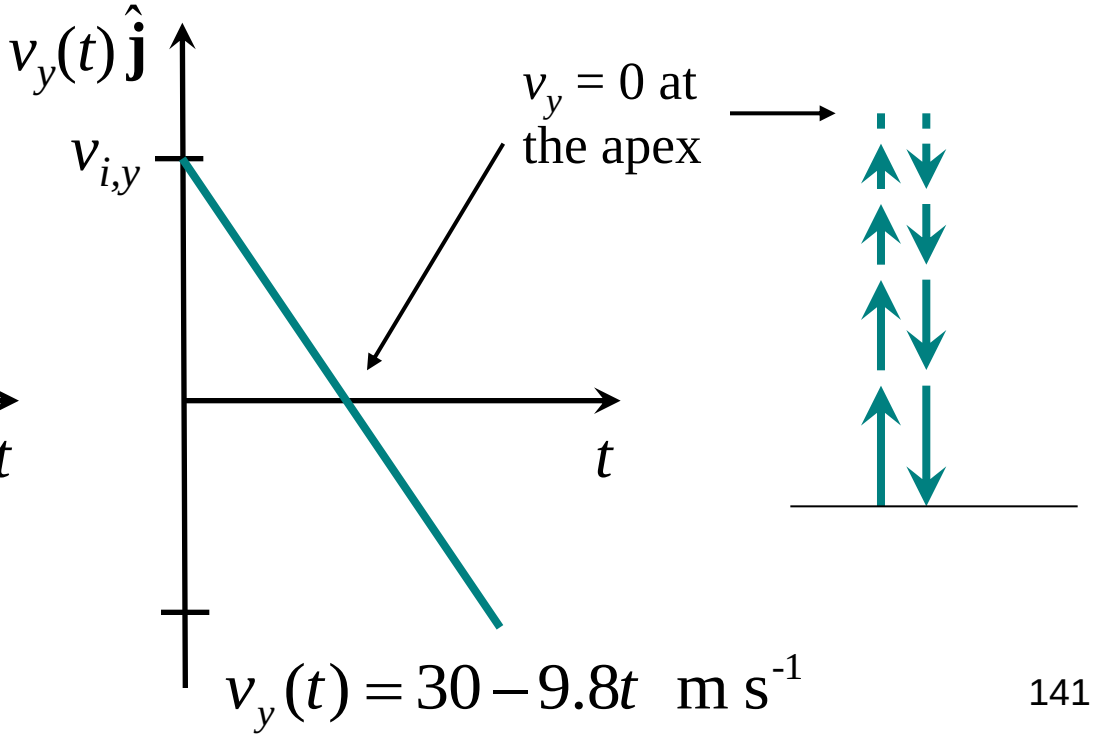
$$x(t) = 40t \text{ m}$$



$$v_{f,x} = 40 \text{ m s}^{-1}$$



$$y(t) = 30t - 4.9t^2 \text{ m}$$



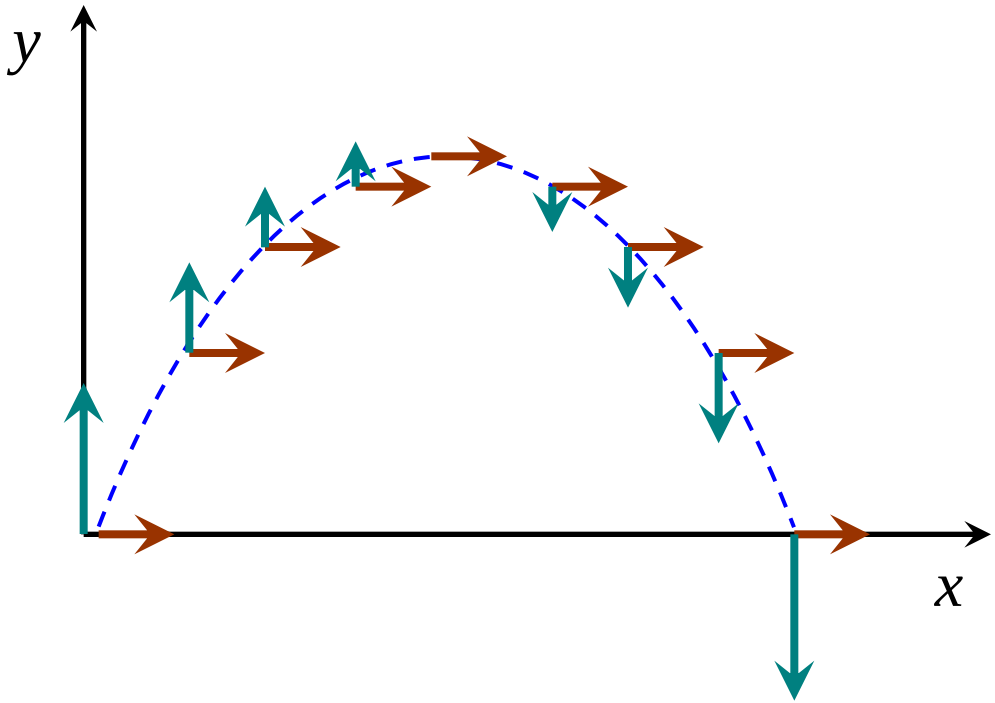
$$v_y(t) = 30 - 9.8t \text{ m s}^{-1}$$

We can think of the motion of a projectile as the combination of a vertical projectile and an object traveling horizontally at a constant velocity.



with

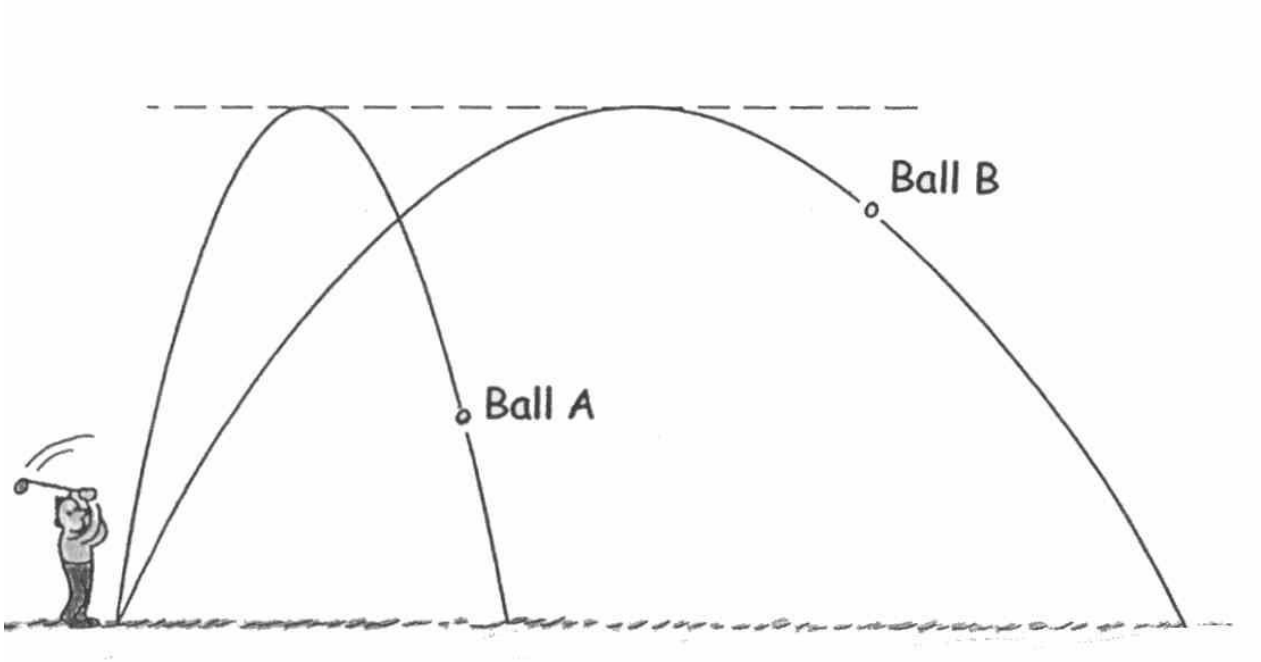




1 2 3 4 5

The figure shows the paths followed by two golf balls, A and B. Does Ball A spend more, the same or less time in the air than Ball B?

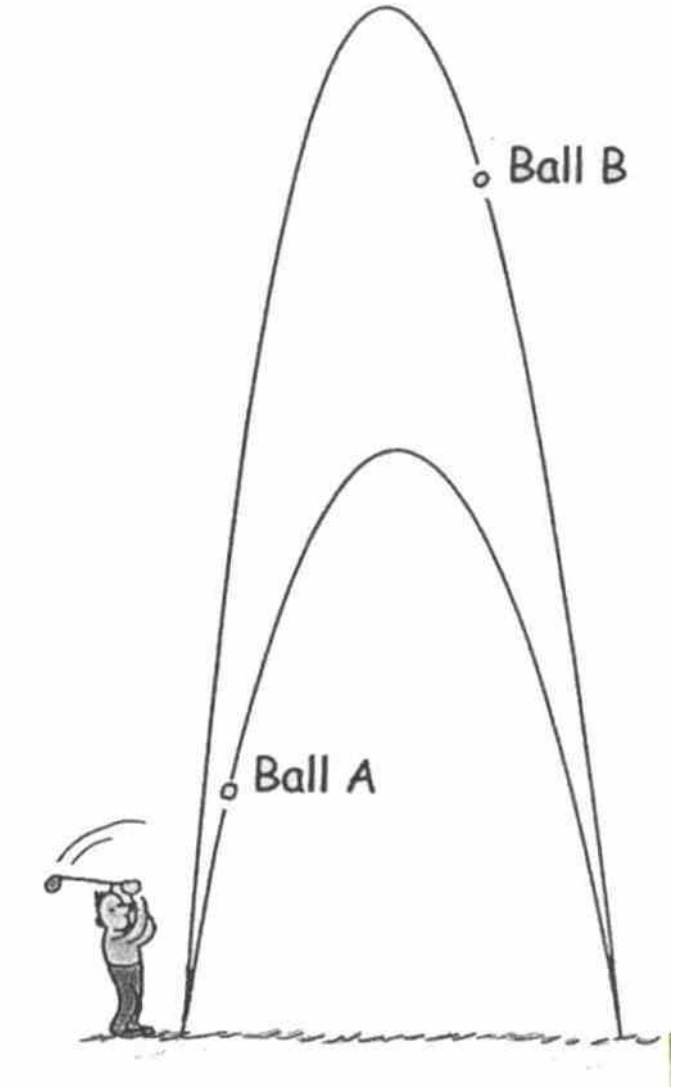
(A) more (B) the same (C) less



1 2 3 4 5

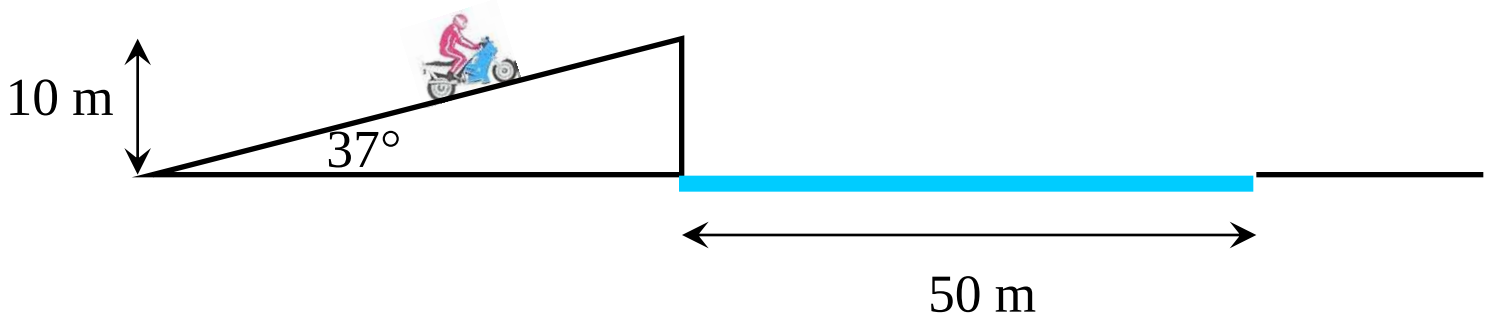
The figure shows the paths followed by two golf balls, A and B.
Does Ball A have a greater, the same or smaller launch speed than Ball B?

(A) greater
(B) the same
(C) smaller



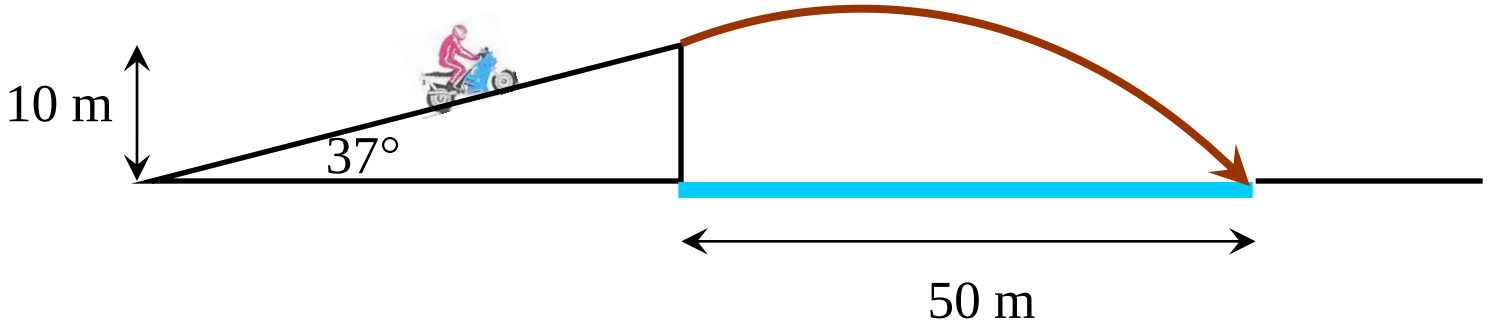
Projectiles: Example 1

Determine the minimum speed that Bugs must have as he leaves the incline on his motorbike in order to just make it across the 50 metre wide swamp.

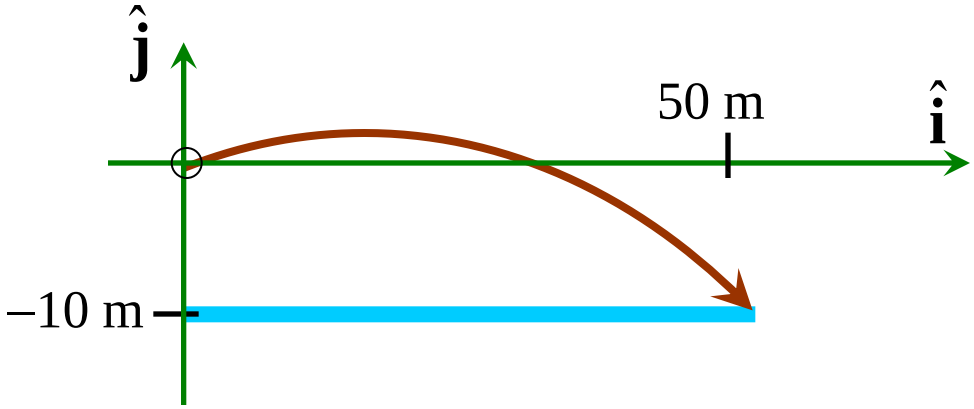


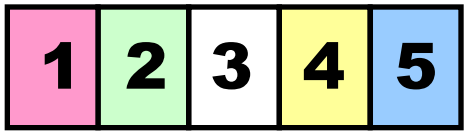
Answer: $v_i = 20.2 \text{ m s}^{-1}$

First draw the path of the projectile (in this case the motorbike) on your diagram.

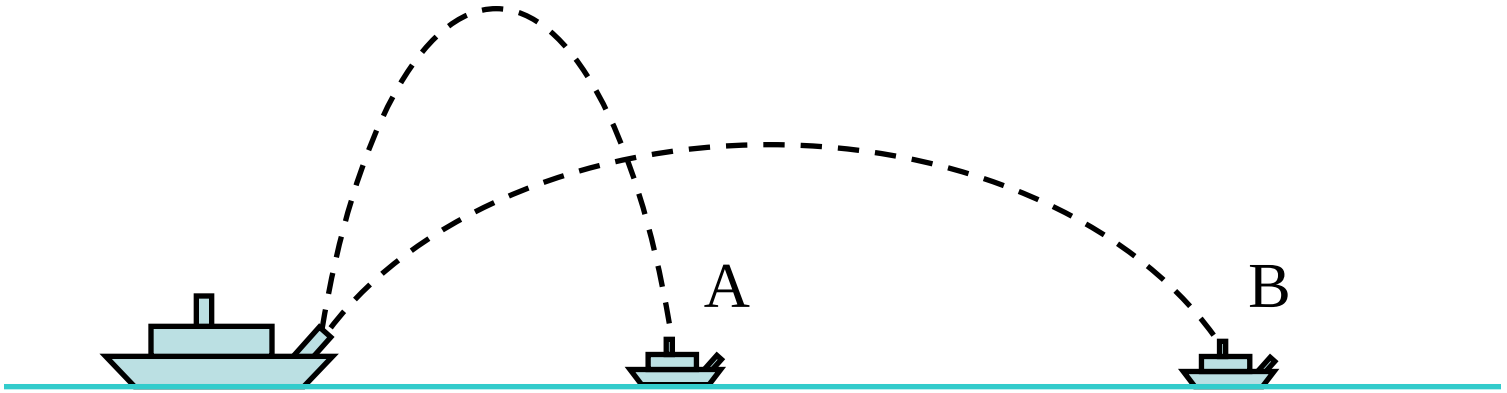


Now draw a set of Cartesian coordinates. Usually it makes sense to set the origin at the initial position of the projectile. Mark the final position on the axes, using symbols for unknowns, if necessary.





A battleship simultaneously fires two shells at the same initial speed at enemy ships. If the shells follow the parabolic trajectories shown, which ship gets hit first?



(A) A
(B) B
(C) both at the same time

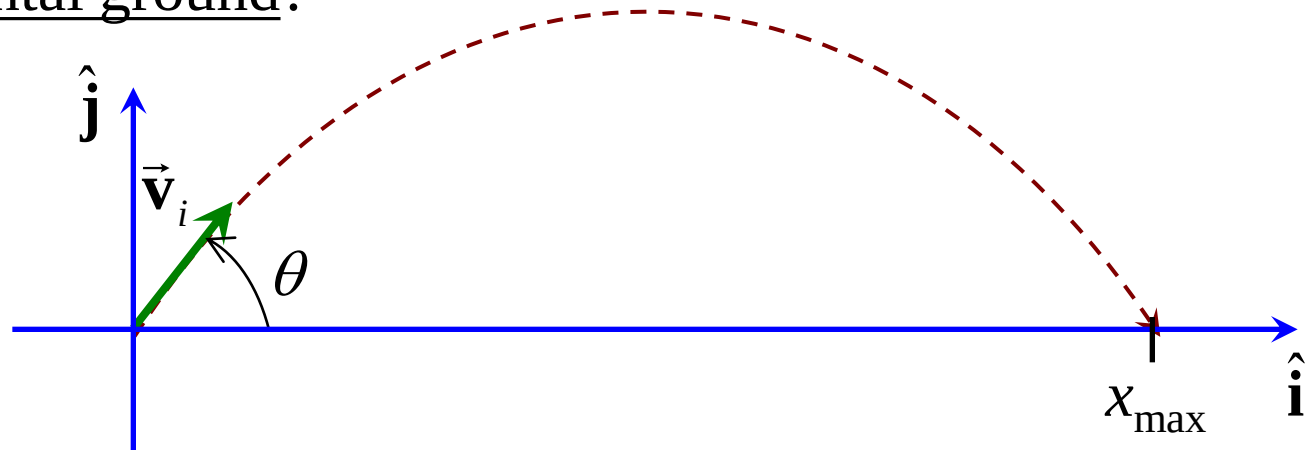
Projectiles: Example 2

You are on the target range preparing to shoot a new rifle when it occurs to you that you would like to know how fast the bullet leaves the gun (the muzzle velocity). You bring the rifle up to shoulder level and aim it *horizontally* at the target centre. Carefully you squeeze off the shot at the target which is 100 m away. When you collect the target you find that your bullet hit 22 cm below where you aimed.

Projectiles: Example 3

You did so well at university you get a job in the airforce as a helicopter pilot. One of the maneuvers that you have to practice is to drop a package from a moving helicopter onto a moving truck on the ground. The difficulty is to know what speed to fly relative to the ground. You are flying horizontally at an altitude of 100 m and you know that when you drop the package, the truck will be 125 m ahead of you (measured along the road) and it will be traveling along the flat road at 60 km h^{-1}.
You estimate the height of the truck above the road to be 3 m.

Something to think about: For a given (fixed) launch velocity $\vec{\mathbf{v}}_i$, what launch angle θ of the projectile will given the largest range on horizontal ground?



For this simple case, our equations for the position are:

$$x_{\max} = (v_i \cos\theta)\Delta t \quad \text{and} \quad 0 = (v_i \sin\theta)\Delta t - \tfrac{1}{2} g \Delta t^2$$

Substituting the one into the other: $x_{\max} = \dfrac{2v_i^2 \cos\theta \sin\theta}{g}$

Giving:

$$x_{\max} = \text{Range}, R = \frac{v_i^2 \sin 2\theta}{g} \qquad \text{since} \quad \sin 2\theta = 2\cos\theta\sin\theta$$

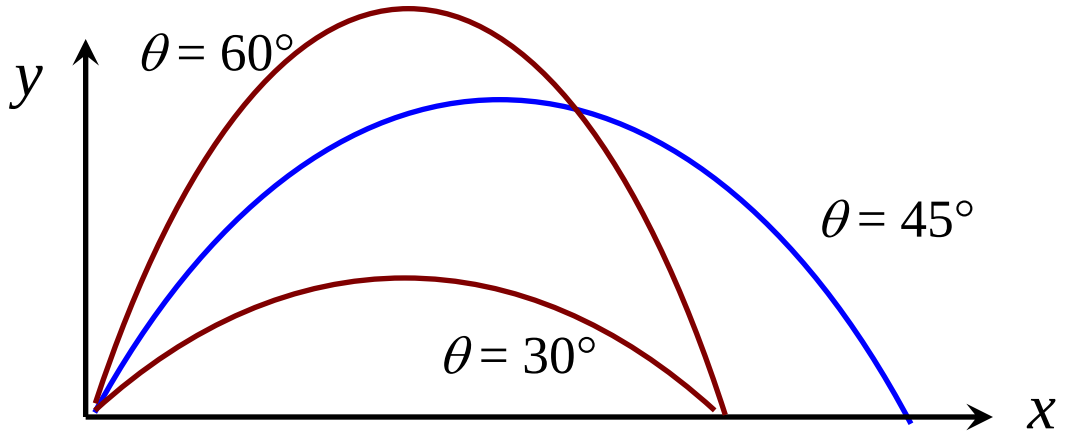
For a projectile on horizontal ground: Range, $R = \dfrac{v_i^2 \sin 2\theta}{g}$

It is clear that for a given v_i, and since g is constant, R is maximum when $\sin 2\theta$ is maximum (= 1),
i.e. when $2\theta = 90°$ or **$\theta = 45°$** .

We can also see that (except for $\theta = 45°$), there are always two angles that give the same R.

For example, $\theta = 30°$ and $\theta = 60°$ will give the same R, since
$\sin(2 \times 30°) = \sin(60°) = \sin(120°) = \sin(2 \times 60°)$

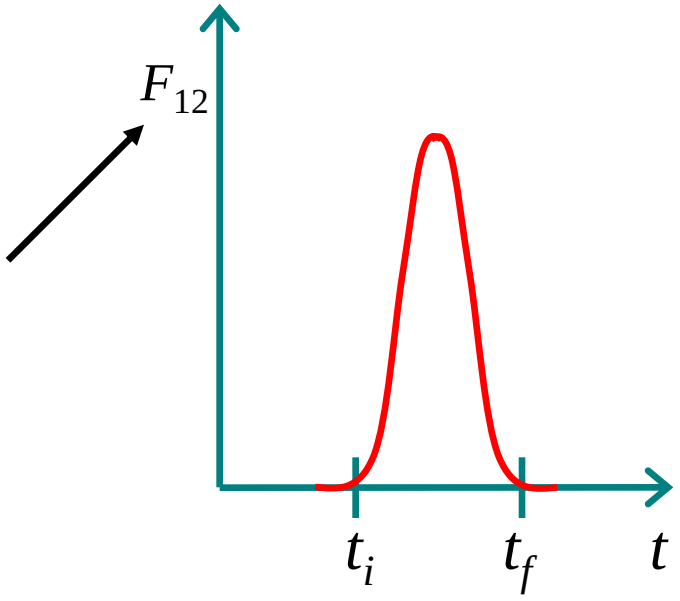
For the same launch speed u:



Estimating interaction times

A collision is when two bodies interact over a short time interval. The forces that the bodies exert on each other are usually so strong during the collision that all forces acting on a body may be ignored.

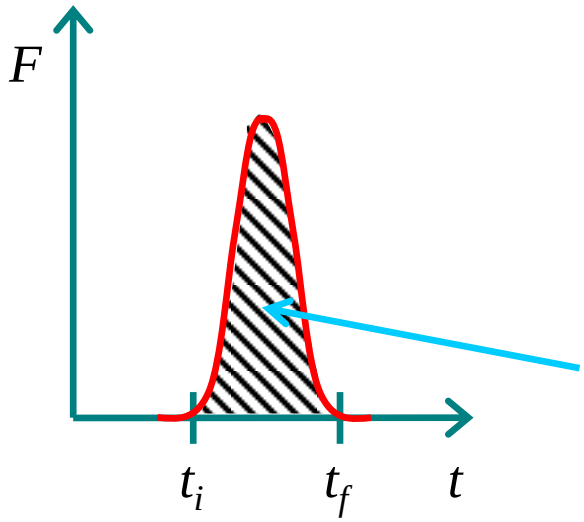
During a collision between two bodies (1 and 2), the contact force exerted by one body on the other jumps from zero to a very large value and then abruptly drops to zero again.



The time interval $\Delta t = t_f - t_i$ is usually very small.

Note that $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$ for the collision

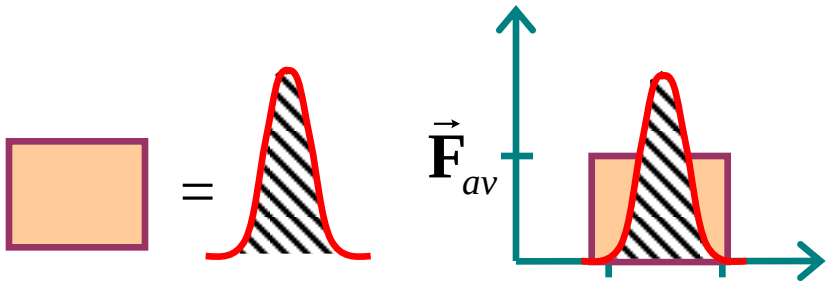
Estimating interaction times



$$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$$

$$\text{Impulse} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \Delta\vec{\mathbf{p}}$$

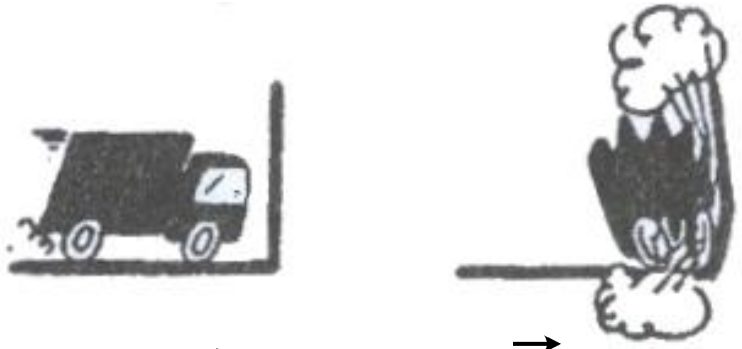
Sometimes it is useful to use the average force $\vec{\mathbf{F}}_{av}$ acting for time Δt to give the same impulse and $\Delta\vec{\mathbf{p}}$.



Then $\vec{\mathbf{F}}_{av}\Delta t = \Delta\vec{\mathbf{p}}$

Estimating interaction times

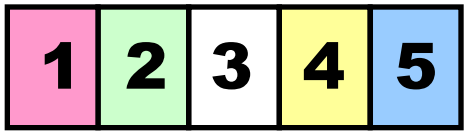
The impulse is the same in each case.



$$m\,\Delta\vec{\mathbf{v}} = \vec{\mathbf{F}}\,\Delta t$$



$$m\,\Delta\vec{\mathbf{v}} = \vec{\mathbf{F}}\,\Delta t$$



Two running students collide head-on.

One student exerts a force of magnitude F on the other student. Suppose we choose BOTH students as the "system" to which to apply the momentum principle. What is the net force acting on this system?

(1) $< F, 0, 0 >$

(2) $< 2F, 0, 0 >$

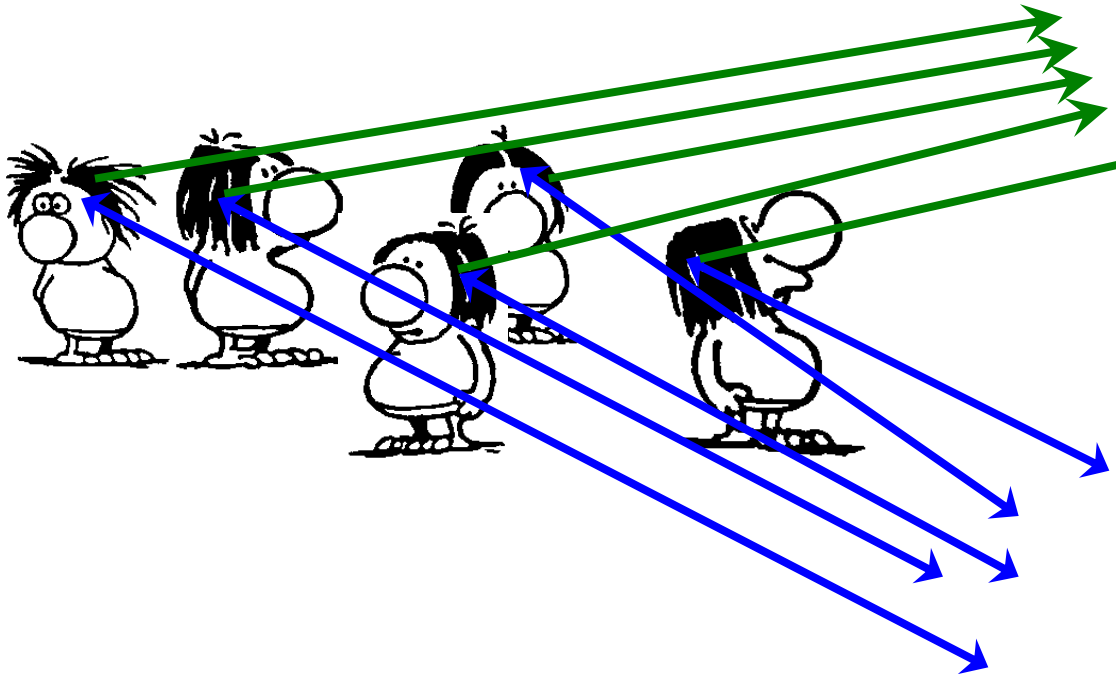
(3) $< 0, 0, 0 >$

When making sense of the ideas in this course, it's useful to think about both the nature of physics and how you learn physics yourself ...

Physical theories
(shared, acontextual)

particularization

Physical model
(shared, contextual)



idealization

Real world
(phenomena)

An approach to solving physics problems

Step 1. Think carefully about the problem situation and draw a picture of what is going on (Pictorial Representation).

- Draw one or more **pictures** which show all the important objects, their motion and any interactions.
- Now ask "**What is being asked**?" "Do I need to calculate something?"
- Think about what **physics concepts** and principles you think will be useful in solving the problem and when they will be most useful.
- Construct a **mental image** of the problem situation - do your friends have the same image?
- Specify a convenient **system** to use - circle this on your picture.
- Identify any **idealisations and constraints** present in the situation - write them down!
- Specify any **approximations** or simplifications which you think will make the problem solution easier, but will not affect the result significantly.

Step 2. Describe the physics (Physics Representation).

- Draw a **coordinate axis** (or axes) onto your picture (decide where to put the origin and on the direction of the axes).
- Translate your pictures into one or more **diagrams** (with axes) which only gives the essential information for a mathematical solution.
- If you are using kinematic concepts, draw a motion diagram specifying the object's velocity and acceleration at definite positions and times.
- If interactions or statics are important, draw idealised, free body and force diagrams.
- When using conservation principles, draw "initial" and "final" diagrams to show how the system changes.
- For optics problems draw a ray diagram.
- For circuit problems, a circuit diagram will be useful.
- Define a **symbol** for every important physics variable in your diagram and write down what information you know (e.g. $T_1 = 30$ N).
- Identify your **target** variable? ("What unknown must I calculate?").

Step 3. Represent the problem mathematically and plan a solution (Mathematical Representation).

- Only now think about what **mathematical expressions** relate the physics variables from your diagrams.
- Using these mathematical expressions, construct specific **algebraic equations** which describe the specific situation above.
- Think about **how** these equations can be combined to find your target variable.
- Begin with an equation that contains the target variable.
- Identify any unknowns in that equation
- Find equations which contain these unknowns
- Do not solve equations numerically at this time.
- Check your equations for **sufficiency**... You have a solution if your plan has as many independent equations are there are unknowns. If not, determine other equations or check the plan to see if it is likely that a variable will cancel from your equations.
- **Plan** the best order in which to solve the equations for the desired variable.

Step 4. Execute the plan

- **Do the algebra** in the order given by your outline.
- When you are done you should have a single equation with your target variable.
- **Substitute** the values (numbers with units) into this final equation.
- Make sure **units** are **consistent** so that they will cancel properly.
- **Calculate** the numerical **result** for the target variable.

Step 5. Evaluate your solution

- Do **vector** quantities have both magnitude and direction ?
- Does the **sign** of your answer make sense ?
- Can someone else follow your solution ? Is it **clear** ?
- Is the **result reasonable** and within your experience ?
- Do the **units** make sense ?

Have you answered the question ?